MA3111: Mathematical Image Processing
Single Image Dehazing

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Outline of "single image dehazing"

In this lecture, we will give a brief introduction to the topics:

- *The atmospheric scattering model.*
- *The dark channel prior-based single image dehazing*

The material of this lecture is based on

- **[HST-2011]** K. He, J. Sun, and X. Tang, Single image haze removal using dark channel prior, *IEEE Transactions on Pattern Analysis and Machine Intelligence,* 33 (2011), pp. 2341-2353.
- **[FLZ-2014]** F. Fang, F. Li, and T. Zeng, Single image dehazing and denoising: a fast variational approach, *SIAM Journal on Imaging Sciences,* 7 (2014), pp. 969-996.

影像去霧演算法研究報告:
https://blog.csdn.net/lhnows/article/details/148194457

Single image dehazing

- Haze/fog is a common atmospheric phenomenon caused by tiny particles (such as dust, smoke, and water droplets) in the air, which reduce visibility and image quality.
- Single-image dehazing is the process of removing haze from a photograph to enhance clarity, contrast, and color accuracy.
- Single image dehazing must estimate haze and scene structure from just one input, *making it ill-posed (no unique solution).*
- The image dehazing can assist computer vision tasks (e.g., object detection, autonomous driving).

hazy image

dehazed image

https://www.cs.huji.ac.il/w~raananf/projects/dehaze_cl/

Histogram equalization (HE, 直方圖均衡化)

- We are given a grayscale image $f:\overline{\Omega}\to[0,1]$. The cumulative histogram *(cumulative distribution function)* T is defined by considering f as a random variable: for $\eta\in[0,1]$, we define

$$\begin{aligned} T(\eta) &:= Prob(f\leq\eta) \\ &= \frac{1}{|\overline{\Omega}|}\left|\{(x,y)\in\overline{\Omega}: f(x,y)\leq\eta\}\right|, \end{aligned}$$

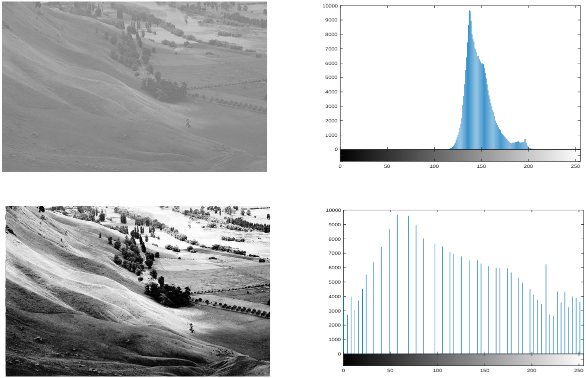
  where $|\cdot|$ denotes the area. *Then $T:[0,1]\to[0,1]$ is a monotonic increasing function.*

- The histogram equalized image $g:\overline{\Omega}\to[0,1]$ is obtained by defining

$$g(x,y):=T(f(x,y)).$$

  Then g is a dehazed image.

Image dehazing: histogram equalized image



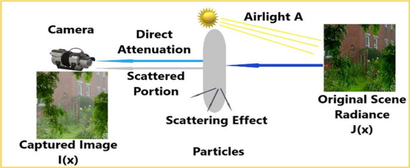
Histogram equalization of 400 $\times$ 600 *image:*
(top) before; (bottom) after; and the corresponding histograms

The atmospheric scattering model

Most image dehazing methods are based on the following atmospheric scattering (AS) model:

$$\boldsymbol{I}(\boldsymbol{x}) = t(\boldsymbol{x})\boldsymbol{J}(\boldsymbol{x}) + (1 - t(\boldsymbol{x}))\boldsymbol{A}, \quad \forall\, \boldsymbol{x} \in \overline{\Omega}. \qquad (\star)$$

- $\boldsymbol{I} = (I_R, I_G, I_B)$ is the observed hazy color image;
- $\boldsymbol{J} = (J_R, J_G, J_B)$ is the haze-free color image to be recovered;
- $\boldsymbol{A} = (A_R, A_G, A_B)$ is the global atmospheric/ambient light (大氣/環境光), a constant vector if the model is homogeneous.
- $t(\boldsymbol{x})$ is the transmission map, representing the portion of light that reaches the camera without being scattered; $0 \le t(\boldsymbol{x}) \le 1$.



The atmospheric scattering model; image credits [Yang and Evans-2021]

Homogeneous atmospheric scattering model

In this lecture, we consider the homogeneous atmospheric scattering model, that is, the atmospheric light $\boldsymbol{A}$ is a constant vector. *Since only the hazy image $\boldsymbol{I}$ is given, we should recover $\boldsymbol{J}$, $t(\boldsymbol{x})$, and $\boldsymbol{A}$ from $\boldsymbol{I}$.*

- If the atmosphere is homogeneous, one usually sets

$$t(\boldsymbol{x}) = e^{-\beta d(\boldsymbol{x})}, \quad \forall\, \boldsymbol{x} \in \overline{\Omega},$$

  where $\beta > 0$ is the medium extinction coefficient (介質消光係數, we can set it as 1) and $d(\boldsymbol{x})$ *is the unknown depth of scene.*
- The estimation of $\boldsymbol{A}$ itself is not so easy. Most of the existing methods set the highest intensity of the input image as $\boldsymbol{A}$.
- We will estimate $t(\boldsymbol{x})$ and $\boldsymbol{A}$ with the help of the so-called *dark channel prior (暗通道先驗).*

Remark: If the atmospheric light factor is a position dependent variable, $\boldsymbol{A} = \boldsymbol{A}(\boldsymbol{x})$, then we have *the non-homogeneous atmospheric scattering model* in the form:

$$\boldsymbol{I}(\boldsymbol{x}) = t(\boldsymbol{x})\boldsymbol{J}(\boldsymbol{x}) + (1 - t(\boldsymbol{x}))\boldsymbol{A}(\boldsymbol{x}), \quad \forall\, \boldsymbol{x} \in \overline{\Omega}.$$

Dark channel prior [HST-2011]

The dark channel prior is a statistical observation: *Most local patches (excluding sky and bright regions) in an outdoor haze-free image $\boldsymbol{J}$ contain some pixels whose intensity is very low in at least one color channel (R, G, or B). In other words,*

$$J^{\text{dark}}(\boldsymbol{x}) \approx 0, \quad \forall\, \boldsymbol{x} \in \overline{\Omega},$$

where the dark channel J^{dark} of an image $\boldsymbol{J} = (J_R, J_G, J_B)$ on $\overline{\Omega}$ is defined as:

$$J^{\text{dark}}(\boldsymbol{x}) = \min_{\boldsymbol{y}\in N(\boldsymbol{x})}\left(\min_{c\in\{R,G,B\}} J_c(\boldsymbol{y})\right) = \min_{c\in\{R,G,B\}}\left(\min_{\boldsymbol{y}\in N(\boldsymbol{x})} J_c(\boldsymbol{y})\right) \quad \forall\, \boldsymbol{x} \in \overline{\Omega},$$

with $N(\boldsymbol{x})$ a local patch centered at $\boldsymbol{x}$.

(a) $\boldsymbol{J}$; (b) for each $\boldsymbol{x}$, minimum of its (R, G, B) values; (c) J^{dark}.

Estimation of the transmission $t(\boldsymbol{x})$

Assume that the transmission $t(\boldsymbol{x})$ is a constant in the local patch $N(\boldsymbol{x})$. By the AS model, $I_c(\boldsymbol{x}) = t(\boldsymbol{x})J_c(\boldsymbol{x}) + (1 - t(\boldsymbol{x}))A_c$, $\forall\, \boldsymbol{x} \in \overline{\Omega}$, $c \in \{R, G, B\}$, we have

$$\min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} I_c(\boldsymbol{y}) = t \min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} J_c(\boldsymbol{y}) + (1-t) \min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} A_c.$$

Since $\boldsymbol{J} = (J_R, J_G, J_B)$ is a haze-free image, by the dark channel prior, we have

$$\min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} J_c(\boldsymbol{y}) = J^{\text{dark}}(\boldsymbol{x}) \approx 0, \quad \forall\, \boldsymbol{x} \in \overline{\Omega}.$$

Since $\boldsymbol{A} = (A_R, A_G, A_B)$ is a constant vector, we obtain

$$\min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} I_c(\boldsymbol{y}) \approx (1-t) \min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} A_c = \min_{c\in\{R,G,B\}} A_c - t \min_{c\in\{R,G,B\}} A_c,$$

which implies that

$$t \min_{c\in\{R,G,B\}} A_c \approx \min_{c\in\{R,G,B\}} A_c - \min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} I_c(\boldsymbol{y}).$$

Therefore,

$$t \approx 1 - \min_{c\in\{R,G,B\}} \min_{\boldsymbol{y}\in N(\boldsymbol{x})} \frac{I_c(\boldsymbol{y})}{A_c}.$$

Estimation of $t(x)$ and the atmospheric light A

- In practice, we take

$$t(\boldsymbol{x}) = 1 - \nu \min_{c \in \{R,G,B\}} \min_{\boldsymbol{y} \in N(\boldsymbol{x})} \frac{I_c(\boldsymbol{y})}{A_c}, \quad \forall\, \boldsymbol{x} \in N(\boldsymbol{x}),$$

where $\nu \in (0, 1]$ is used to keep some depth information of the natural image and is commonly set as $\nu = 0.95$.

(1) If $\nu = 0 \Rightarrow t(\boldsymbol{x}) = 1 \Rightarrow d(\boldsymbol{x}) = -\ln(t(\boldsymbol{x})) = 0$, unreasonable!

(2) If $t(\boldsymbol{x}) = 0 \Rightarrow d(\boldsymbol{x}) = \infty$, unreasonable!

- **Estimation of A:** Among the pixels which are *the brightest* 0.1% *in* I^{dark}, the pixels with the highest intensity in I_c are picked as the atmospheric light A_c.
- After obtaining $t(\boldsymbol{x})$ and $\boldsymbol{A}$, a dehazed image can be recovered according to the AS model $(\star)$:

$$\boldsymbol{J}(\boldsymbol{x}) = \frac{\boldsymbol{I}(\boldsymbol{x}) - \boldsymbol{A}}{\max\{t(\boldsymbol{x}), \varepsilon\}} + \boldsymbol{A}, \quad \forall\, \boldsymbol{x} \in N(\boldsymbol{x}),$$

where $\varepsilon > 0$ (e.g., $\varepsilon = 0.1$) is a constant to avoid division by zero.

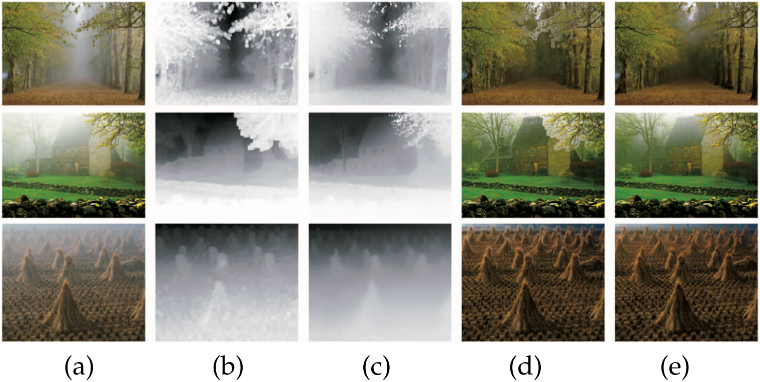
Artifacts around edges

In general, the resulting image $J(\boldsymbol{x})$ may contain some artifacts around edges because the transmission $t(\boldsymbol{x})$ is not so accurate.

hazy image *dehazed image*

In order to overcome this defect, the authors of [HST-2011] use the soft matting method to refine $t(\boldsymbol{x})$.

Some numerical results in [HST-2011]



Haze removal: (a) input hazy images. (b) estimated transmission maps before soft matting; (c) refined transmission maps after soft matting; (d) and (e) recovered images using (b) and (c), respectively.