# MA 2007B: Linear Algebra I – Quiz #8

**Name:** **Student ID number:**

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(1) Let $\boldsymbol{V}$ be a subspace of $\mathbb{R}^n$ and $\boldsymbol{V}^\perp := \{\boldsymbol{x} \in \mathbb{R}^n |\ \boldsymbol{x} \cdot \boldsymbol{v} = 0,\ \forall \boldsymbol{v} \in \boldsymbol{V}\}$ be the orthogonal complement of $\boldsymbol{V}$. Show that $\boldsymbol{V}^\perp$ is also a subspace of $\mathbb{R}^n$.

**Proof:**

- Let $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{V}^\perp$. Then $\boldsymbol{x} \cdot \boldsymbol{v} = 0$ and $\boldsymbol{y} \cdot \boldsymbol{v} = 0$, $\forall\ \boldsymbol{v} \in \boldsymbol{V}$.
  $\therefore\ (\boldsymbol{x} + \boldsymbol{y}) \cdot \boldsymbol{v} = \boldsymbol{x} \cdot \boldsymbol{v} + \boldsymbol{y} \cdot \boldsymbol{v} = 0,\ \forall\ \boldsymbol{v} \in \boldsymbol{V}$
  $\therefore\ \boldsymbol{x} + \boldsymbol{y} \in \boldsymbol{V}^\perp$
- Let $\boldsymbol{x} \in \boldsymbol{V}^\perp$ and $\alpha \in \mathbb{R}$. Then $\boldsymbol{x} \cdot \boldsymbol{v} = 0$, $\forall\ \boldsymbol{v} \in \boldsymbol{V}$.
  $\therefore\ (\alpha \boldsymbol{x}) \cdot \boldsymbol{v} = \alpha(\boldsymbol{x} \cdot \boldsymbol{v}) = 0,\ \forall\ \boldsymbol{v} \in \boldsymbol{V}$
  $\therefore\ \alpha \boldsymbol{x} \in \boldsymbol{V}^\perp$

$\therefore\ \boldsymbol{V}^\perp$ is a subspace of $\mathbb{R}^n$

(2) Let $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{a} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Find the projection $\boldsymbol{p}$ of $\boldsymbol{b}$ onto the line through $\boldsymbol{a}$ and find the projection matrix $\boldsymbol{P}$.

**Solution:**

- The projection of $\boldsymbol{b}$ onto the line through $\boldsymbol{a}$ is given by

$$\boldsymbol{p} = \frac{\boldsymbol{a}^\top \boldsymbol{b}}{\boldsymbol{a}^\top \boldsymbol{a}} \boldsymbol{a} = \frac{5}{9} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{9} \\ \frac{10}{9} \\ \frac{10}{9} \end{bmatrix}.$$

- The projection matrix is given by

$$\boldsymbol{P} = \frac{\boldsymbol{a}\boldsymbol{a}^\top}{\boldsymbol{a}^\top \boldsymbol{a}} = \frac{1}{9} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 4 \\ 2 & 4 & 4 \end{bmatrix}.$$