MA 2007B: Linear Algebra I – Quiz #6

Name: **Student ID number:**

Let $\boldsymbol{A}$ be the 3×4 matrix,

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & 8 & 10 \\ 3 & 3 & 10 & 13 \end{bmatrix}.$$

(1) Find the reduced row echelon form $\boldsymbol{R}$ of matrix $\boldsymbol{A}$ and indicate the pivot columns.

Solution:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 2 & 8 & 10 \\ 3 & 3 & 10 & 13 \end{bmatrix} \xrightarrow{(\ell_{21}=2,\ \ell_{31}=3)} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 4 & 4 \end{bmatrix} \xrightarrow{(\ell_{33}=1)} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{(row2/4)} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{(row1-2\times row2)} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = rref(\boldsymbol{A}) := \boldsymbol{R}.$$

The first and third columns of $\boldsymbol{A}$ (also of $\boldsymbol{R}$) are pivot columns.

(2) Find two special solutions $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ of $\boldsymbol{Ax} = \boldsymbol{0}$ by the standard technique.

Solution: Note that $\boldsymbol{Ax} = \boldsymbol{0} \iff \boldsymbol{Rx} = \boldsymbol{0}$.

By (1), we know the free variables are x_2 and x_4.

Let $x_2 = 1$ and $x_4 = 0$. Then we have $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$.

$\Longrightarrow x_1 = -1$ and $x_3 = 0$. We obtain a special solution

$$\boldsymbol{s}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Let $x_2 = 0$ and $x_4 = 1$. Then we have $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$.

$\Longrightarrow x_1 = -1$ and $x_3 = -1$. We obtain a special solution

$$\boldsymbol{s}_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \end{bmatrix}.$$