MA 2007B: Linear Algebra I – Quiz #3

Name: **Student ID number:**

---

(1) (3 points) Express $\boldsymbol{AB}$ in terms of columns times rows ("outer product") and then find the result,

$$\boldsymbol{AB} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} = ?$$

Solution:

$$\begin{aligned} \boldsymbol{AB} &= \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 3 & 0 \\ 6 & 6 & 0 \\ 6 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 4 & 8 & 4 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 0 \\ 10 & 14 & 4 \\ 7 & 8 & 1 \end{bmatrix}. \end{aligned}$$

(2) (3 points) Find the inverse $\boldsymbol{E}^{-1}$ of the elementary matrix:

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Solution:

$\because$ the elementary matrix $\boldsymbol{E}$ subtracts 5 times row 1 from row 2

$\therefore$ its inverse $\boldsymbol{E}^{-1}$ adds 5 times row 1 to row 2

$$\therefore\ \boldsymbol{E}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3) (4 points) Let $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ be three $n \times n$ matrices. Show that if $\boldsymbol{BA} = \boldsymbol{I}$ and $\boldsymbol{AC} = \boldsymbol{I}$, then $\boldsymbol{B} = \boldsymbol{C}$.

Proof:

$\because$ $(\boldsymbol{BA})\boldsymbol{C} = \boldsymbol{B}(\boldsymbol{AC})$ and $\boldsymbol{BA} = \boldsymbol{I}$ and $\boldsymbol{AC} = \boldsymbol{I}$

$\therefore$ $\boldsymbol{IC} = \boldsymbol{BI}$

$\therefore$ $\boldsymbol{C} = \boldsymbol{B}$