MA 2007B: Linear Algebra I – Quiz #2

Name: **Student ID number:**

Consider the following system of linear equations $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$:

$$\begin{cases} 2x - 3y &= 3, \\ 4x - 5y + z &= 7, \\ 2x - y - 3z &= 5. \end{cases}$$

(1) (5 points) Apply elimination and back substitution to solve the linear system.

Solution:

$$\begin{cases} 2x & -3y & & = & 3, \\ 4x & -5y & +z & = & 7, \\ 2x & -y & -3z & = & 5. \end{cases} \implies \ell_{21} = 2, \ell_{31} = 1, \quad \begin{cases} 2x & -3y & & = & 3, \\ & y & +z & = & 1, \\ & 2y & -3z & = & 2. \end{cases}$$

$$\implies \ell_{32} = 2, \quad \begin{cases} 2x & -3y & & = & 3, \\ & y & +z & = & 1, \\ & & -5z & = & 0. \end{cases} \implies z = 0 \implies y = 1 \implies x = 3.$$

The solution is $(x, y, z) = (3, 1, 0)$.

(2) (2 points) Find the three pivots p_1, p_2, p_3 and the three multipliers ℓ_{21}, ℓ_{31}, ℓ_{32}.

Solution:

Three pivots: $p_1 = 2, \; p_2 = 1, \; p_3 = -5.$

Three multipliers: $\ell_{21} = 2, \; \ell_{31} = 1, \; \ell_{32} = 2.$

(3) (3 points) Let $\boldsymbol{U}$ be the resulting upper triangular matrix of the elimination and let $\boldsymbol{L}$ be the lower triangular matrix defined by

$$\boldsymbol{L} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}.$$

Check that $\boldsymbol{L}\boldsymbol{U} = \boldsymbol{A}$.

Solution:

$$\boldsymbol{L}\boldsymbol{U} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 0 \\ 4 & -5 & 1 \\ 2 & -1 & -3 \end{bmatrix} = \boldsymbol{A}.$$