MA 1002B Calculus: Quiz 2

Class: **Score:**

Student Number:

Name:

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(1). (5 points) Show that $\sum_{n=1}^{\infty} \frac{1}{1+n^2}$ converges by the integral test.

Solution:

Let $f(x) = \frac{1}{1+x^2}$.

Then f is continuous, positive and decreasing for $x \geq 1$, and

$\int_1^{\infty} f(x)dx = \int_1^{\infty} \frac{1}{1+x^2}dx = \lim_{b\to\infty} \int_1^b \frac{1}{1+x^2}dx = \lim_{b\to\infty} \tan^{-1} x \Big]_1^b$

$= \lim_{b\to\infty} \left(\tan^{-1} b - \tan^{-1} 1\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$.

By the integral test, $\sum_{n=1}^{\infty} f(n) = \sum_{n=1}^{\infty} \frac{1}{1+n^2}$ converges!

(2). (5 points) Show that $\sum_{n=1}^{\infty} \frac{1+n\ln n}{n^2+5}$ diverges by the limit comparison test.

Solution:

Let $a_n = \frac{1+n\ln n}{n^2+5}$ and $b_n = \frac{1}{n}$.

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n+n^2\ln n}{n^2+5} = \lim_{n\to\infty} \frac{\frac{1}{n}+\ln n}{1+\frac{5}{n^2}} = \infty$.

$\because \sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series)

$\therefore$ By the limit comparison test, $\sum_{n=1}^{\infty} \frac{1+n\ln n}{n^2+5}$ diverges!