MA 8019: Numerical Analysis I
Syllabus and Introduction

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Syllabus

- **Instructor:** Prof. Suh-Yuh Yang (楊肅煜)
  - Office: M315, Hong-Jing Hall
  - Phone: 03-4227151 ext. 65130
- **Office hours**: Tuesday 10:00 $\sim$ 12:00 am or by appointment. No teaching assistant!
- **Prerequisites**: Calculus, Linear Algebra and some knowledge of the software MATLAB: https://portal.ncu.edu.tw/
  校園授權軟體服務網裡面有關於Matlab的下載方式說明！
- **Assignments**: approximately every two weeks, will consist of theoretical problems or computer projects. The students are encouraged to discuss homework with other classmates. *Direct copying is absolutely not allowed.*
- **Examinations:** there will be *a midterm and a final exam.*
- **Grading policy:** *assignments 40%, midterm 30% and final 30%.*

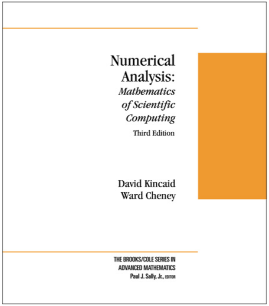
Course objectives

(1) This course introduces students to various types of mathematical analysis that are commonly needed in scientific computing.

(2) The subject of numerical analysis is treated from a mathematical point of view, offering a complete analysis of methods for scientific computing with appropriate motivations and careful proofs.

Textbook

David Kincaid and Ward Cheney, *Numerical Analysis: Mathematics of Scientific Computing, Third Edition,* 2002, Brooks/Cole.



http://www.ma.utexas.edu/CNA/NA3/index.html

Errata: http://www.ma.utexas.edu/CNA/NA3/errata.html

Important dates

- The period for adding and dropping: September 4-18, 2024
- The period for withdrawing: October 21-November 29, 2024
- Moon Festival: September 17 (Tue), 2024, *no class!*
- Midterm: November 5 (Tue), 2024 (9th week)
- Sports Day: November 20 (Wed), 2024, *no class!*
- Final exam: December 31 (Tue), 2024 (17th week)

What is numerical analysis?

- **Numerical analysis – mathematics of scientific computing:** it involves the study, development and analysis of algorithms (procedures) for obtaining numerical solutions to various mathematical problems.
- **Scientific computing:** solving mathematical problems numerically on the computer (methods/constructive proofs $\to$ algorithms $\to$ codes $\to$ display).
- **Problem modeling**
  (1) physical phenomena: too expensive to perform all tests with prototypes.
  (2) mathematical model: (differential or integral equations) too complex or very difficult for paper/pencil solution.
  (3) computational model: (numerical methods) approximation of mathematical model.

Outline of the course

This semester will cover the following topics:

- Mathematical preliminaries (Chapter 1)
- Computer arithmetic (Chapter 2)
- Solving nonlinear equations (Chapter 3)
- Systems of linear equations (Chapter 4)
- Solving eigenvalue problems (Chapter 5)

Next semester (MA 8020: Numerical Analysis II):

- Interpolation and approximation (Chapter 6)
- Differentiation and integration (Chapter 7)
- Numerical ordinary differential equations (Chapter 8)
- Numerical partial differential equations (Chapter 9)

Topic 1: Mathematical preliminaries

- **Taylor's Theorem:** for functions in single or several variables.
- **Order of convergence:** big O and little o notations.

Topic 2: Computer arithmetic

Real number system: $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}^c$

- $\mathbb{N}$: the set of natural numbers (positive integers), $\{1, 2, 3, \cdots\}$.
- $\mathbb{Z}$: the set of integers, $\{0, 1, -1, 2, -2, \cdots\}$.
- $\mathbb{Q}$: the set of rational numbers, $\{1.1, 3.14, 2/3, \cdots\}$.
- $\mathbb{Q}^c := \mathbb{R} \setminus \mathbb{Q}$: the set of irrational numbers, $\{\pi = 3.1415926535897..., e = 2.718281828..., \sqrt{2} = 1.4142..., \cdots\}$.

Questions:

- How to represent the numbers?
- How to perform the basic operations +, −, *, / ?
- What is the error?

Topic 3: Solving nonlinear equations

Question: given a function $f : \mathbb{R} \to \mathbb{R}$, find a point $x^* \in \mathbb{R}$ such that

$$f(x^*) = 0.$$

- If $f(x)$ is simple, such as $f(x) = 3x + 1$ or $f(x) = 3x^2 - 4x + 1$, then one can use the root formulas. In general, one has to find the root(s) numerically.
- We will study
  (1) iterative methods for finding the root (bisection method, secant method, Newton type methods),
  (2) convergence of the methods,
  (3) extension to nonlinear systems.

Topic 4: Solving linear systems of equations ($Ax = b$)

Linear system: find the vector $[x_1, x_2]^\top$ such that

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}.$$

The size of the problem is $n = 2$.

- For small n, the system can be solved by hand, but for large n (could be as large as $n = 10^6$), one has to use computers.
- We will study

  (1) vector, matrix, norm,
  (2) Gaussian elimination and matrix factorizations,
  (3) iterative methods,
  (4) error analysis,
  (5) algorithm complexity.

Topic 5: Solving eigenvalue problems ($Ax = \lambda x$)

Eigenvalue problem: find a number $\lambda \in \mathbb{C}$ and a vector $\mathbf{0} \neq [x_1, x_2]^\top \in \mathbb{C}^2$ such that

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

The size of this problem is $n = 2$.

- For small n, the system can be solved by hand, but for large n (could be as large as $n = 10^6$), one has to use computers.
- We will study
  1. power method,
  2. Schur's and Gershgorin's theorems,
  3. orthogonal decompositions,
  4. QR-algorithm.