小波理論與應用 MA5110

Homework Assignment 2

Due Oct. 31. 2024

Problem 1. Prove Theorem 6.9 in the note:

Theorem 6.9. *If ψ and ϕ are wavelets and f, g are functions which belong to $L^2(\mathbb{R})$, then*

(i) (Linearity) *For any scalars α and β,*

$$W_\psi[\alpha f + \beta g] = \alpha W_\psi[f] + \beta W_\psi[g].$$

(ii) (Translation) *With T_c denoting the translation operator defined by $(T_c f)(t) = f(t-c)$,*

$$W_\psi[T_c f](a,b) = W_\psi[f](a, b-c) \qquad \textit{and} \qquad W_{T_c\psi}[f](a,b) = W_\psi[f](a, b+ca).$$

(iii) (Dilation) *For $c > 0$, with D_c denoting the (scaled) dilation operator defined by $(D_c f)(t) = \frac{1}{\sqrt{c}} f\Big(\frac{t}{c}\Big)$,*

$$W_\psi[D_c f](a,b) = \frac{1}{\sqrt{c}} W_\psi[f]\Big(\frac{a}{c}, \frac{b}{c}\Big) \qquad \textit{and} \qquad W_{D_c\psi}[f](a,b) = \frac{1}{\sqrt{c}} W_\psi[f](ac, b).$$

(iv) (Symmetry) *For any $a \neq 0$,*

$$W_\psi[f](a,b) = W_f[\psi]\Big(1, -\frac{b}{a}\Big).$$

(v) (Parity) *With P denoting the parity operator defined by $(Pf)(t) = f(-t)$ (that is, $P = D_{-1}$),*

$$W_{P\psi}[Pf](a,b) = W_\psi[f](a, -b).$$

(vi) (Anti-linearity) *For any scalars α, β,*

$$W_{\alpha\psi+\beta\phi}[f] = \overline{\alpha} W_\psi[f] + \overline{\beta} W_\phi[g].$$