Vector Analysis MA2014-* Midterm Exam 2

National Central University, Dec. 5 2018

Problem 1. (15%) Evaluate $\displaystyle\int_{[0,a]\times[0,b]} e^{\max\{b^2x^2,a^2y^2\}}\,d(x,y)$, where a, b are positive numbers.

Problem 2. Complete the following.

1. (15%) Sketch the solid whose volume is given by the sum of the iterated integrals

$$\int_0^6\int_{\frac{z}{2}}^3\int_{\frac{z}{2}}^y dxdydz + \int_0^6\int_3^{\frac{12-z}{2}}\int_{\frac{z}{2}}^{6-y} dxdydz\,.$$

2. (15%) Write the volume as a single iterated integral in the order $dydzdx$ and find the volume of the solid.

Problem 3. Let T be the trapezoid with vertices $(1,1)$, $(2,2)$, $(2,0)$ and $(4,0)$. Evaluate the integral $\displaystyle\int_T e^{(y-x)/(y+x)}d(x,y)$

1. (15%) by transforming to polar coordinates, and

2. (10%) by using the transformation $u = y - x$ and $v = y + x$.

Problem 4. (15%) Show that if $\lambda > \frac{1}{2}$, there does not exist a real-valued continuous function u such that for all x in the closed interval $[0,1]$, $u(x) = 1 + \lambda\displaystyle\int_x^1 u(y)u(y-x)\,dy$.

Problem 5. (15%) Find the volume of the region of points (x,y,z) such that $(x^2+y^2+z^2+8)^2 \leqslant 36(x^2+y^2)$.

挑戰題：

Problem 6. (10%) Let A be the area of the region in the first quadrant bounded by the line $y = \frac{1}{2}x$, the x-axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$. Find the positive number m such that A is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the y-axis, and the ellipse $\frac{1}{9}x^2 + y^2 = 1$.