# Exercise Problem Sets 9

May. 14. 2021

**Problem 1.** Consider the heat equation

$$\begin{aligned} \frac{\partial u}{\partial t}(x,t) &= \alpha^2 \frac{\partial^2 u}{\partial x^2} + q(x) && 0 < x < L\,, t > 0\,, \\ u(0,t) = u(L,t) &= 0 && t > 0\,, \\ u(x,0) &= f(x) && 0 < x < L\,. \end{aligned}$$

What do you expect from the solution by passing to the limit as $t \to \infty$? Explain your answer.

**Problem 2.** Solve the heat equation

$$\begin{aligned} \frac{\partial u}{\partial t}(x,t) &= \alpha^2 \frac{\partial^2 u}{\partial x^2} + q(x,t) && 0 < x < L\,, t > 0\,, \\ u_x(0,t) = u_x(L,t) &= 0 && t > 0\,, \\ u(x,0) &= f(x) && 0 < x < L\,, \end{aligned}$$

by the methodology that we talked about in class. Under the assumption that $q = 0$, what do you expect from the solution by passing to the limit as $t \to \infty$? How about if $q(x,t) = g(x)$ for some $g$ satisfying $\displaystyle\int_0^L g(x)\,dx = 0$? Explain your answers.

**Problem 3.** Solve the heat equation

$$\begin{aligned} \frac{\partial u}{\partial t}(x,t) &= \alpha^2 \frac{\partial^2 u}{\partial x^2} + q(x,t) && 0 < x < L\,, t > 0\,, \\ u_x(0,t) = u(L,t) &= 0 && t > 0\,, \\ u(x,0) &= f(x) && 0 < x < L\,, \end{aligned}$$

by the methodology that we talked about in class. Under the assumption that $q = 0$, what do you expect from the solution by passing to the limit as $t \to \infty$? Explain your answer.