Exercise Problem Sets 7

May. 2. 2021

Problem 1. 1. Find the eigenfunctions v of $\dfrac{d^2}{dx^2}$ on $[-p,p]$ with the boundary condition $v(-p) = v(p) = 0$; that is, find non-trivial v satisfying

$$v''(x) = \lambda v(x) \quad \forall\, x \in [-p,p]\,, \qquad v(-p) = v(p) = 0 \tag{$\star$}$$

for some constant $\lambda \in \mathbb{R}$.

2. Suppose that $\{v_k\}_{k=1}^{\infty}$ are collection of eigenfunctions satisfying $(\star)$ which forms a complete orthogonal set on $[-\pi,\pi]$ (that is, $p=\pi$ in $(\star\star)$). For a given function f defined on $[-\pi,\pi]$, f can be expressed as

$$f = \sum_{k=1}^{\infty} c_k v_k\,.$$

Find the "linear combination" of $\{v_k\}_{k=1}^{\infty}$ that is used to represent $f(x) = x^2$.

Problem 2. 1. Find the eigenfunctions w of $\dfrac{d^2}{dx^2}$ on $[0,p]$ with the boundary condition $w(0) = w'(p) = 0$; that is, find non-trivial w satisfying

$$w''(x) = \lambda w(x) \quad \forall\, x \in [0,p]\,, \qquad w(0) = w'(p) = 0 \tag{$\star\star$}$$

for some constant $\lambda \in \mathbb{R}$.

2. Suppose that $\{w_k\}_{k=1}^{\infty}$ are collection of eigenfunctions satisfying $(\star\star)$ which forms a complete orthogonal set on $[0,\pi]$ (that is, $p=\pi$ in $(\star\star)$). For a given function f defined on $[0,\pi]$, f can be expressed as

$$f = \sum_{k=1}^{\infty} c_k w_k\,.$$

Find the "linear combination" of $\{w_k\}_{k=1}^{\infty}$ that is used to represent $f(x) = x^2$.

Problem 3. 1. Find the cosine series of the function $f : [0,\pi] \to \mathbb{R}$ given by $f(x) = \sin x$.

2. Find the sine series of the function $f : [0,\pi] \to \mathbb{R}$ given by $f(x) = \cos x$.