Exercise Problem Sets 2

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Problem 1. Is it possible that $\boldsymbol{X}_1(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t$ and $\boldsymbol{X}_2(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} te^t$ is a fundamental set of a linear system $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} a(t) & b(t) \\ c(t) & d(t) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$? Explain your answer using the Wronskian.

Problem 2. Solve the linear system $\boldsymbol{X}' = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boldsymbol{A}\boldsymbol{X}$ following the steps

Step 1: Find second order differential equations that x_1 and x_2 satisfy;

Step 2: Find the general solutions of x_1 and x_2;

Step 3: Find the relations among coefficients of the general solutions of x_1 and x_2;

Step 4: Find a fundamental set of solutions to $\boldsymbol{X}' = \boldsymbol{A}\boldsymbol{X}$;

that we talked about in class to find a fundamental set of the linear system $\boldsymbol{X}' = \boldsymbol{A}\boldsymbol{X}$ for following given $\boldsymbol{A}$:

(1) $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. (2) $\boldsymbol{A} = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$. (3) $\boldsymbol{A} = \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$.

(4) $\boldsymbol{A} = \begin{bmatrix} -5 & 5 \\ 3 & -3 \end{bmatrix}$. (5) $\boldsymbol{A} = \begin{bmatrix} 5 & -5 \\ 3 & -3 \end{bmatrix}$.

Problem 3. Find a fundamental set of the linear system $\boldsymbol{X}' = \begin{bmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \boldsymbol{X}$ following the steps given in Problem 2.