Exercise Problem Sets 1

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Problem 1. Write the linear system

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ 8 \end{bmatrix}\sin t + \begin{bmatrix} t-4 \\ 2t+1 \end{bmatrix} e^{4t}$$

without the use of matrices.

Hint: x and y both satisfies second-order scalar differential equations.

Problem 2. Let $\boldsymbol{A}(t) = \begin{bmatrix} a_{11}(t) & a_{12}(t) & \cdots & a_{1n}(t) \\ a_{21}(t) & a_{22}(t) & \cdots & a_{2n}(t) \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}(t) & a_{n2}(t) & \cdots & a_{nn}(t) \end{bmatrix}$ and $\boldsymbol{F}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ \vdots \\ f_n(t) \end{bmatrix}$ be matrices having continuous entries, and $\boldsymbol{\Phi}(t)$ be a fundamental matrix of the homogeneous system $\boldsymbol{X}' = \boldsymbol{A}(t)\boldsymbol{X}$. Show that if $\boldsymbol{X}_p(t) = \boldsymbol{\Phi}(t)\,\boldsymbol{U}(t)$, where $\boldsymbol{U}(t) = \big[u_1(t), u_2(t), \cdots, u_n(t)\big]^{\mathrm{T}}$, is a particular solution of the non-homogeneous system $\boldsymbol{X}' = \boldsymbol{A}(t)\boldsymbol{X} + \boldsymbol{F}(t)$, then $\boldsymbol{U}$ satisfies that

$$\boldsymbol{\Phi}(t)\,\boldsymbol{U}'(t) = \boldsymbol{F}(t)\,.$$