

# Differential Equations MA2041-A Final Exam

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In the following problems, when you obtain a “candidate” of solutions to a specific ODE or IVP, verify that it is indeed a solution. Each verification counts 5pts.

**Problem 1.** Complete the following.

1. Given a solution  $y = \varphi_1(t) = t^2$  to

$$t^2(t+3)y''' - 3t(t+2)y'' + 6(1+t)y' - 6y = 0 \quad \text{for } t > 0, \quad (0.1)$$

find a fundamental set of the ODE above.

2. Find the solution to the IVP

$$t^2(t+3)y''' - 3t(t+2)y'' + 6(1+t)y' - 6y = t+3, \quad y(0) = y'(0) = y''(0) = 0.$$

**Problem 2.** Solve the initial value problem

$$y'' - 4y' + 4y = t^2 e^t, \quad y(0) = y'(0) = 0$$

using

1. the method of undetermined coefficients.
2. the method of variation of parameters.
3. the method of annihilator.
4. the Laplace transform.

**Problem 3.** Find the Laplace transform of the function  $f(t) = t^n e^{at}$ .

**Problem 4.** Complete the following.

1. Show that if  $f$  is piecewise continuous and of exponential order  $\alpha$  for some  $\alpha$ , then  $\mathcal{L}(f)$ , the Laplace transform of  $f$ , is bounded for  $s > 2\alpha$ .
2. Find the solution to the initial value problem

$$y'' + 3ty' - 6y = 1, \quad y(0) = y'(0) = 0.$$