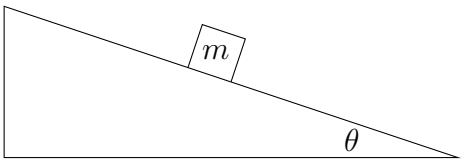
Differential Equations Recommended Exercise 1

Problem 1. 推導放置在角度為 θ 的斜面上、懸於平行於斜面虎克常數為 k 的彈簧上質量為 m 的物體的運動方程式。



Problem 2. In class we derive the differential equation for the brachistochrone curve connecting $(0,0)$ and (a,b), where $b<0$:

$$\Big[\frac{f'(y)}{\sqrt{-2gy}\sqrt{1+f'(y)^2}}\Big]' = 0\,, \quad f(0)=0, f(a)=b\,.$$

Solve this differential equation.

Suppose that the brachistochrone curve connecting $(0,0)$ and (a,b) can be represented as $y=h(x)$ (thus $h(0)=0$ and $h(a)=b$). Use the variational principle to derive the differential equation that h has to satisfy.

Problem 3. Suppose that there exists a twice continuously differentiable minimizer $y=y(t)$ to the following variational problem

$$\min_{\varphi\in\mathcal{A}} \int_0^a L(\varphi,\varphi',t)\,dt\,, \text{ where } \mathcal{A}=\big\{\varphi:[0,a]\to\mathbb{R}\,\big|\,\varphi(0)=\varphi(a)=0\big\}\,,$$

where $L(p,q,t)$ is differentiable with respect to p and q. Derive the equation that the minimizer y has to satisfy.