# 數值分析 MA-3021

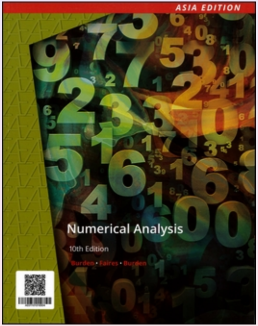
# Syllabus

- 授課老師: 鄭經斅 (Ching-hsiao Arthur Cheng)
  - 研究室：M301 校分機：65111
  - Email: cchsiao@math.ncu.edu.tw
  - Office hour: 星期三 1PM 至 2PM
- 課程網頁：

  http://www.math.ncu.edu.tw/~cchsiao/Course/Numerical_Analysis_082
- 助教：蘇逸鎮 / 研究室：M115 / 校分機：65109
- 課程預備知識：微積分、線性代數，以及寫程式的能力（本課程使用 matlab）
- 作業：原則上每兩週出一次作業，作業請至課程網頁下載。
- 評分方式：作業 40%、期中考 30%、期末考 30%。

# Textbook

Richard L. Burden and J. Douglas Faires, *Numerical Analysis, 10th Edition*, Thomson Brooks/Cole, 2016. (東華書局暨新月書局代理)



# Scientific Computing v.s. Numerical Analysis

- **Problem modelling**
  - physical phenomena: too expensive to perform all tests with prototypes.
  - mathematical model (differential or integral equations): too complex or very difficult for paper/pencil solution.
  - computational model (numerical methods): approximation of mathematical model.
- **Scientific computing:** solving mathematical problems numerically on the computer (methods/constructive proofs → algorithms → codes → display).
- **Numerical analysis – mathematics of scientific computing:** it involves the study, development and analysis of algorithms (procedures) for obtaining numerical solutions to various mathematical problems.

# Scientific Computing v.s. Numerical Analysis

- **Scientific computing: interdisciplinary (跨學科)**

  science/engineering;

  numerical analysis;

  computer science;

  software engineering.

# Topics covered in this course

- Mathematical preliminaries
- Solutions of nonlinear equations
- Interpolation and polynomial approximation
- Numerical differentiation and integration
- Direct and iterative methods for solving linear systems
- Numerical ordinary differential equations*
- Numerical partial differential equations*

- **Review of Calculus/Analysis.**
- **Taylor's Theorem:** for functions in single or several variables.
- **Rate of convergence:** big $O$ notation.

# Solutions of nonlinear equations

- **Question:** given a function $f : \mathbb{R} \to \mathbb{R}$. Find a point $x^* \in \mathbb{R}$ such that

$$f(x^*) = 0.$$

- If $f(x)$ is simple, such as $f(x) = 3x + 1$ or $f(x) = 3x^2 - 4x + 1$, then one can use the root formulas. In general, one has to find the root(s) numerically.
- We will study
  - iterative methods for finding the root (bisection method, secant method, Newton type methods);
  - convergence of the methods;
  - extension to systems of nonlinear equations.

- **Polynomial interpolation (多項式插值)**

  We are given $n+1$ data points $(x_i, y_i)$, $i = 0, 1, \cdots, n$, and we seek a polynomial $p$ such that $p(x_i) = y_i$, $0 \leqslant i \leqslant n$, where $y_i = f(x_i)$ for some function $f$.

- **Hermite interpolation** the interpolation of a function and some of its derivatives at a set of nodes. e.g., find a polynomial $p$ such that $p(x_i) = f(x_i)$ and $p'(x_i) = f'(x_i)$, $i = 0, 1$.

- **Spline (樣條) interpolation**

  A spline function of degree $k$ is a piecewise polynomial of degree at most $k$ having continuous derivatives of all orders up to $k-1$.

# Numerical differentiation and integration

- **Numerical differentiation**
  - Based on Taylor's theorem: $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(\xi)$.
  - Based on polynomial interpolation: let $p$ be the Lagrange interpolation of $f$. Then $f'(x) \approx p'(x)$.
- **Numerical integration based on interpolation:** let $p$ be the Lagrange interpolation of $f$. Then $\int_a^b f(x)dx \approx \int_a^b p(x)dx$.
- **Gaussian quadrature (高斯積分法):** find $A_i$ and $x_i$, $i = 0, 1, \cdots, n$, such that $\int_a^b f(x)dx \approx \sum_{i=0}^{n} A_i f(x_i)$ and it will be exact for polynomials of degree $\leqslant 2n+1$.

# Direct and iterative methods for solving linear systems

**Linear system:** find the vector $(x_1, x_2)^\top$ such that

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}.$$

The size of the problem is $n = 2$. For small $n$, the system can be solved by hand, but for large $n$ (could be as large as $n = 10^6$), one has to use computers. We will study

- vector, matrix, norm
- Gaussian elimination and matrix factorizations
- iterative methods
- error analysis

# Numerical ordinary differential equations

- **Existence and uniqueness theory** of the initial value problem:
$$x'(t) = f(t,x), \quad x(t_0) = x_0.$$
- **Taylor-series method:**
$$x(t+h) = x(t) + hx'(t) + \frac{h^2}{2!}x''(t) + \frac{h^3}{3!}x'''(t) + \cdots$$
- **Runge-Kutta methods:** in Taylor-series method, we have to determine $x'', x''', \cdots$. The Runge-Kutta methods avoid this difficulty.
- **Multistep methods:** e.g., Adams-Bashforth-formula of order $5$:
$$x_{n+1} = x_n + \frac{h}{720}\{1901f_n - 2774f_{n-1} + 2616f_{n-2} - 1274f_{n-3} + 251f_{n-4}\}.$$
- **Convergence, stability and consistency:** for multistep method, *convergent* $\Longleftrightarrow$ *stable* + *consistent*.
- **Boundary value problems:** finite difference methods.

# Numerical partial differential equations

- **Parabolic problems:** finite difference method (explicit, implicit)
- **Elliptic problems:** finite difference & finite element methods
- **Hyperbolic problems:** characteristics