# Exercise Problem Sets 6

Jun. 12 2020

(due Jun. 17 2020)

**Problem 1.** Consider solving the initial value problem

$$x' = f(t,x) \equiv t^{-2}(tx - x^2)\,, \qquad x(1) = 2$$

using numerical method. Note that the exact solution to the IVP above is $x(t) = \big(\frac{1}{2} + \ln t\big)^{-1} t$. Complete the following.

(1) Use the second order Runge-Kutta method

$$\begin{aligned} k_1 &= f(t_n, x_n)\,, \\ k_2 &= f(t_n + h, x_n + hk_1)\,, \\ x_{n+1} &= x_n + \frac{h}{2}(k_1 + k_2)\,, \end{aligned}$$

with $h = 1/128$, $1/256$ and $1/512$ to compute approximated values of $x(3)$. Plot the numerical solution versus the exact solution.

(2) Find the global truncation error at $t = 3$ and maximal local truncation errors for each $h$. Examine numerically that the second order Runge-Kutta method is an order one method.

(3) Use the fourth order Runge-Kutta method

$$\begin{aligned} k_1 &= f(t_n, x_n)\,, \\ k_2 &= f\big(t_n + \frac{h}{2}, x_n + \frac{h}{2}k_1\big)\,, \\ k_3 &= f\big(t_n + \frac{h}{2}, x_n + \frac{h}{2}k_2\big)\,, \\ k_4 &= f\big(t_n + h, x_n + hk_3\big)\,, \\ x_{n+1} &= x_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)\,, \end{aligned}$$

with $h = 1/128$, $1/256$ and $1/512$ to compute approximated values of $x(3)$. Plot the numerical solution versus the exact solution.

(4) Find the global truncation error at $t = 3$ and maximal local truncation errors for each $h$. Examine numerically that the fourth order Runge-Kutta method is an order one method.