Fourier Analysis MA3019 Final Exam

National Central University, 2016

Problem 1. Let $f(x) = e^{-s|x|^2}$ and $g(x) = e^{-t|x|^2}$. Use the inversion formula to compute $f * g$.

Problem 2. Find the Fourier transform of the function $f(x) = xe^{tx^2}$ for $t < 0$.

Problem 3. Let $f(x) = \chi_{(0,\infty)}(x)e^{-tx}$; that is,

$$f(x) = \begin{cases} e^{-tx} & \text{if } x > 0\,, \\ 0 & \text{if } x \leqslant 0\,. \end{cases}$$

Find the Fourier transform of f for $t > 0$.

Problem 4. Show that a function $f \in L^2(\mathbb{R}^n)$ is real if and only if $\widehat{f}(-\xi) = \overline{\widehat{f}(\xi)}$.

Problem 5. Let $\mathscr{S}(\mathbb{R}^n, \mathbb{R}^n)$ denote the collection of vector-valued functions $u : \mathbb{R}^n \to \mathbb{R}^n$ whose components are Schwartz functions. Show the Korn inequality:

$$\sum_{i,j=1}^{n} \Big\| \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \Big\|^2_{L^2(\mathbb{R}^n)} \geqslant 2 \sum_{i,j=1}^{n} \Big\| \frac{\partial u_i}{\partial x_j} \Big\|^2_{L^2(\mathbb{R}^n)} \qquad \forall\, u \in \mathscr{S}(\mathbb{R}^n, \mathbb{R}^n)\,.$$

Hint: Use the Plancherel formula/identity.