數學流體力學之理論與計算

Homework Assignment 4

Due date: Prob.1-3 on Nov. 16.

Part I: Theoretical assignments

Problem 1. Show that there exists a generic constant C such that

$$\|uv\|_{H^3(\mathbb{T}^3)} \le C\Big[\|u\|_{L^\infty(\mathbb{T}^3)}\|v\|_{H^3(\mathbb{T}^3)} + \|u\|_{H^3(\mathbb{T}^3)}\|v\|_{L^\infty(\mathbb{T}^3)}\Big] \quad \forall\, u, v \in H^3(\mathbb{T}^3)\,.$$

Problem 2. Define the notion of weak solution to the following elliptic equation

$$u - \Delta u = f \qquad \text{in} \quad \mathbb{T}^n\,,$$

and show that there exists a unique weak solution $u \in H^1(\mathbb{T}^n)$ for all $f \in L^2(\mathbb{T}^n)$.

Part II: Computational assignments

Problem 3. Suppose that "position" and a (discrete) vector field "u_val" (which is defined on each intersection point and time independent) are given. Write a matlab® program "plot_trajectory" with inputs "starting_point" and "end_time" so that the output plots the trajectory of the function $x(t)$, where $x(t)$ solves

$$\begin{aligned} x'(t) &= u(x(t)) && \text{in} \quad [0, \text{end\_time}]\,,\\ x(0) &= \text{starting\_point} \end{aligned}$$

and u is the linear interpolation of the discrete vector field u_val.