微積分 MA1004-I 班加分項目一

Due Mar. 10. 2025

Problem 1. In class we have talked about the Dirichlet test for the convergence of a series stated below:

> Let $\{a_n\}_{n=1}^\infty$, $\{p_n\}_{n=1}^\infty$ be sequences of real numbers such that
>
> (i) the sequence of partial sums of the series $\sum_{k=1}^\infty a_k$ is bounded; that is, there is $M \in \mathbb{R}$ such that $\left|\sum_{k=1}^n a_k\right| \leqslant M$ for all $n \in \mathbb{N}$.
>
> (ii) $\{p_n\}_{n=1}^\infty$ is a decreasing sequence, and $\lim_{n\to\infty} p_n = 0$.
>
> Then $\sum_{k=1}^\infty a_k p_k$ converges.

1. Show that the series $\sum_{k=1}^\infty \frac{\sin(kx)}{k}$ converges for all $x \in \mathbb{R}$.
2. Use computer (e.g. matlab) to help you decide where the series above converges to (for each x at which the series converges). Plot what you get!

Hint:

1. Try to use the formula

$$\sum_{k=1}^n \sin kx = \frac{\cos(n+\frac{1}{2})x - \cos\frac{x}{2}}{2\sin\frac{x}{2}}$$

   as long as $\sin\frac{x}{2} \neq 0$.