Exercise Problem Sets 13

May 24. 2024

Problem 1. Rewrite the following iterated integrals as an equivalent iterated integral in the five other orders.

(1) $\displaystyle\int_0^1 \Big[\int_y^1 \Big(\int_0^y f(x,y,z)\,dz \Big) dx \Big] dy$

(2) $\displaystyle\int_0^1 \Big[\int_y^1 \Big(\int_0^z f(x,y,z)\,dx \Big) dz \Big] dy$

(3) $\displaystyle\int_0^1 \Big[\int_0^{1-x^2} \Big(\int_0^{1-x} f(x,y,z)\,dy \Big) dz \Big] dx$

(4) $\displaystyle\int_0^3 \Big[\int_0^x \Big(\int_0^{9-x^2} f(x,y,z)\,dz \Big) dy \Big] dx$

(5) $\displaystyle\int_0^1 \Big[\int_{\sqrt{x}}^1 \Big(\int_0^{1-y} f(x,y,z)\,dz \Big) dy \Big] dx$

(6) $\displaystyle\int_{-1}^1 \Big[\int_{x^2}^1 \Big(\int_0^{1-y} f(x,y,z)\,dz \Big) dy \Big] dx$

(7) $\displaystyle\int_0^1 \Big[\int_{x^2}^{\sqrt{x}} \Big(\int_{x^2}^y f(x,y,z)\,dz \Big) dy \Big] dx$

Problem 2. Evaluate the following iterated integrals.

(1) $\displaystyle\int_0^1 \Big(\int_{\arcsin y}^{\frac{\pi}{2}} \cos x \sqrt{1+\cos^2 x}\,dx \Big) dy$

(2) $\displaystyle\int_{-5}^5 \Big[\int_0^{\sqrt{25-x^2}} \Big(\int_0^{\frac{1}{x^2+y^2}} \sqrt{x^2+y^2}\,dz \Big) dy \Big] dx$

(3) $\displaystyle\int_0^4 \Big[\int_0^1 \Big(\int_{2y}^y \frac{2\cos(x^2)}{\sqrt{z}}\,dx \Big) dy \Big] dz$

(4) $\displaystyle\int_0^1 \Big[\int_0^1 \Big(\int_{x^2}^1 xz \exp(zy^2)\,dy \Big) dx \Big] dz$

(5) $\displaystyle\int_0^1 \Big[\int_{\sqrt[3]{z}}^1 \Big(\int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2}\,dx \Big) dy \Big] dz$

(6) $\displaystyle\int_0^2 \Big[\int_0^{4-x^2} \Big(\int_0^x \frac{\sin(2z)}{4-z}\,dy \Big) dz \Big] dx$

Problem 3. Find volume of the solid that lies under $z = x^2 + y^2$ and above the region R in the xy-plane bounded by the line $y = 2x$ and parabola $y = x^2$.

Problem 4. Evaluate the triple integral $\displaystyle\iiint_D dV$, where D is bounded by $z = x^2 + y^2$, $x^2 + y^2 = 4$ and $z = 0$.