Calculus MA1001-B Quiz 12

National Central University, Dec. 24 2019

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Problem 1. (2pts) Show at least two ways to find $\int \sec^4 x \tan^3 x \, dx$.

Solution. Observing that $\sec^2 x \, dx = d(\tan x)$,

$$\int \sec^4 x \tan^3 x \, dx = \int (\tan^2 + 1) \tan^3 x \, d(\tan x) = \frac{\tan^6 x}{6} + \frac{\tan^4 x}{4} + C \, .$$

On the other hand, noting that $d(\sec x) = \sec x \tan x \, dx$,

$$\int \sec^4 x \tan^3 x \, dx = \int \sec^3 x (\sec^2 x - 1) \, d(\sec x) = \frac{\sec^6 x}{6} - \frac{\sec^4 x}{4} + C \, .$$ □

Problem 2. (3pts) Find the indefinite integral $\int \exp(\sqrt{x}) \, dx$.

Solution. Let $u = \sqrt{x}$. Then $u^2 = x$ (so that $dx = 2u \, du$); thus

$$\int e^{\sqrt{x}} \, dx = \int e^u \cdot 2u \, du = 2 \int u e^u \, du = 2\Big[u e^u - \int e^u \, du\Big] = 2(u-1)e^u + C = 2(\sqrt{x} - 1)e^{\sqrt{x}} + C \, .$$ □

Problem 3. (2pts) Find the indefinite integral $\int \sqrt{x} \arctan \sqrt{x} \, dx$.

Solution. Let $u = \arctan \sqrt{x}$ and $v = \dfrac{2x^{\frac{3}{2}}}{3}$ (so that $dv = \sqrt{x} dx$). Then

$$\begin{aligned}
\int \sqrt{x} \arctan \sqrt{x} \, dx &= \frac{2x^{\frac{3}{2}} \arctan \sqrt{x}}{3} - \frac{2}{3} \int \frac{x^{\frac{3}{2}}}{1 + \sqrt{x}^2} \cdot \frac{1}{2\sqrt{x}} \, dx \\
&= \frac{2x^{\frac{3}{2}} \arctan \sqrt{x}}{3} - \frac{1}{3} \int \frac{x}{1+x} \, dx = \frac{2x^{\frac{3}{2}} \arctan \sqrt{x}}{3} - \frac{1}{3} \int \Big(1 - \frac{1}{1+x}\Big) \, dx \\
&= \frac{2x^{\frac{3}{2}} \arctan \sqrt{x}}{3} - \frac{1}{3} x + \frac{1}{3} \ln(1+x) + C \, .
\end{aligned}$$ □

Problem 4. (3pts) Write the rational function $\dfrac{2x^2}{(x+1)^2(x^2+1)}$ as the sum of partial fractions.

Solution. Using partial fractions, $\dfrac{2x^2}{(x+1)^2(x^2+1)} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{Cx+D}{x^2+1}$. First $B = \dfrac{2(-1)^2}{(-1)^2+1} = 1$; thus

$$\frac{A}{x+1} + \frac{Cx+D}{x^2+1} = \frac{2x^2}{(x+1)^2(x^2+1)} - \frac{1}{(x+1)^2} = \frac{x^2-1}{(x+1)^2(x^2+1)} = \frac{x-1}{(x+1)(x^2+1)} \, .$$

Having the equation above, $A = \dfrac{-1-1}{(-1)^2+1} = -1$. Therefore,

$$\frac{Cx+D}{x^2+1} = \frac{x-1}{(x+1)(x^2+1)} + \frac{1}{x+1} = \frac{x - 1 + x^2 + 1}{(x+1)(x^2+1)} = \frac{x}{x^2+1}$$

which shows that $C = 1$ and $D = 0$. As a consequence,

$$\frac{2x^2}{(x+1)^2(x^2+1)} = \frac{-1}{x+1} + \frac{1}{(x+1)^2} + \frac{x}{x^2+1} \, .$$ □