Calculus MA1001-A Midterm 2 Sample

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Problem 1. Suppose that the limit $\lim_{n\to\infty} \frac{n^\alpha r^n (2n)!}{(n!)^2}$ exists and is non-zero. Find α and r.

Problem 2. Find all $p \in \mathbb{R}$ such that $\sum_{k=3}^{\infty} \frac{\ln(1+k) - \ln k}{(\ln k)^p \ln(\ln k)}$ converges. Note that you need to provide the reason for the convergence or divergence of the power series for each p.

Problem 3. Show that $\sum_{k=1}^{\infty} \frac{(-1)^k \cos(kx)}{k}$ converges for all $x \in \mathbb{R}$.

Problem 4. Find the radius of convergence and the interval of convergence of the power series

$$\sum_{k=2}^{\infty} \frac{x^{2k}}{k(\ln k)^2} .$$

Problem 5. Suppose that $x(t)$ is a function of t satisfying the following equations

$$x''(t) - 2x'(t) + 2x(t) = 0 , \quad x(0) = 0 , \quad x'(0) = 1 ,$$

where $'$ denotes the derivatives with respect to t.

1. Assume that the function $x(t)$ can be written as a power series (on a certain interval), that is, $x(t) = \sum_{k=0}^{\infty} a_k t^k$. Find a_0, a_1, $\cdots$, a_5.
2. Show that the 5-th Maclaurin polynomial of $e^t \sin t$ agrees with the 5-th Maclaurin polynomial of $x(t)$.

Problem 6. Use the Taylor Theorem to show that

$$\ln(1+x^2) \leqslant x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4} + \frac{x^{10}}{5} \qquad \forall\, x \in \mathbb{R} .$$

Problem 7. Find n such that

$$\left| \cos 1 - \sum_{k=0}^{n} \frac{(-1)^k}{(2k)!} \right| < 5 \times 10^{-6} .$$

Explain your answer.