Calculus MA1002-A Quiz 09

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Problem 1. (5%) Evaluate the double integral $\iint\limits_R xy\,dA$, where R is the region bounded by line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Solution. Solving for the points of intersection of the given line and parabola, we find that $(x-1)^2 = 2x + 6$; thus $x = 5$ or $x = -1$ which implies that the points of intersection is $(5, 4)$ and $(-1, -2)$. Therefore, the region R can be expressed as $R = \big\{(x, y)\,\big|\, -2 \leqslant y \leqslant 4, \frac{y^2-6}{2} \leqslant x \leqslant y + 1\big\}$, and the Fubini Theorem implies that

$$\begin{aligned}
\iint\limits_R xy\,dA &= \int_{-2}^{4}\Big(\int_{\frac{y^2-6}{2}}^{y+1} xy\,dx\Big)dy = \int_{-2}^{4}\Big(\frac{x^2y}{2}\Big|_{x=\frac{y^2-6}{2}}^{x=y+1}\Big)dy = \frac{1}{2}\int_{-2}^{4}\Big[(y+1)^2y - \big(\frac{y^2-6}{2}\big)^2 y\Big]dy \\
&= \frac{1}{2}\int_{-2}^{4}\Big[(y+1)^3 - (y+1)^2 - \frac{1}{4}(y^2-6)^2 y\Big]dy \\
&= \frac{1}{2}\Big[\frac{(y+1)^4}{4} - \frac{(y+1)^3}{3} - \frac{1}{24}(y^2-6)^3\Big]\Big|_{y=-2}^{y=4} \\
&= \frac{1}{2}\Big[\frac{625}{4} - \frac{125}{3} - \frac{1000}{24} - \frac{1}{4} + \frac{-1}{3} - \frac{1}{3}\Big] \\
&= \frac{1}{2}\Big[156 - \frac{125}{3} - \frac{125}{3} - \frac{2}{3}\Big] = \frac{1}{2}\cdot\frac{216}{3} = 36\,.
\end{aligned}$$

□

Problem 2. (5%) Find the surface area of the part of the surface $z = x^2 + 2y$ that lies above the triangular region T in the xy-plane with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$.

Solution. Note that the triangle T can be expressed as $T = \big\{(x, y)\,\big|\, 0 \leqslant x \leqslant 1, 0 \leqslant y \leqslant x\big\}$. Therefore, the surface area of interest is

$$\begin{aligned}
\iint\limits_T \sqrt{1 + f_x(x,y)^2 + f_y(x,y)^2}\,dA &= \int_0^1\Big(\int_0^x \sqrt{1 + 4x^2 + 4}\,dy\Big)dx = \int_0^1 x\sqrt{5 + 4x^2}\,dx \\
&= \frac{1}{12}(5 + 4x^2)^{\frac{3}{2}}\Big|_{x=0}^{x=1} = \frac{1}{12}\big(9^{\frac{3}{2}} - 5^{\frac{3}{2}}\big) = \frac{1}{12}(27 - 5\sqrt{5})\,.
\end{aligned}$$

□