

Extra Exercise Problem Sets 7

Feb. 22 2019

Problem 1. In this problem we intend to compute

$$\int_0^{\frac{\pi}{2}} \ln \sin x \, dx .$$

1. Determine whether the integral above is an improper integral or not.

2. Prove the identities

$$\int_0^{\pi} \ln \sin x \, dx = 2 \int_0^{\frac{\pi}{2}} \ln \sin(2x) \, dx$$

and

$$\int_0^{\frac{\pi}{2}} \ln \sin x \, dx = \int_0^{\frac{\pi}{2}} \ln \cos x \, dx .$$

3. Find $\int_0^{\frac{\pi}{2}} \ln \sin x \, dx$ (using identities in 2).

Problem 2. In this problem you might need the *comparison test for improper integrals* in problem 52 of Section 8.8 in the book. Determine whether the following improper integrals converge or not.

$$1. \int_0^{\infty} \frac{dx}{\sqrt[3]{x^4 - x^2}}. \quad 2. \int_1^{\infty} \frac{dx}{x(\ln x)^\alpha} \text{ for } \alpha > 0. \quad 3. \int_{100}^{\infty} \frac{dx}{x(\ln \ln x)^\alpha} \text{ for } \alpha > 0.$$