Extra Exercise Problem Set 12

Apr. 27 2019

Problem 1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} x^2 \arctan \dfrac{y}{x} - y^2 \arctan \dfrac{x}{y} & \text{if } xy \neq 0\,, \\ 0 & \text{if } xy = 0\,. \end{cases}$$

Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$.

Problem 2. Investigate the differentiability of the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x+y^2} & \text{if } x + y^2 \neq 0\,, \\ 0 & \text{if } x + y^2 = 0\,. \end{cases}$$

Problem 3. Let $f : (a,b) \to \mathbb{R}$ be a function of one variable, and f is differentiable at $c \in (a,b)$.

1. Show that if $g(x,y) = f(x)$, then g is differentiable at (c,d) for all $d \in \mathbb{R}$.
2. Show that if $h(x,y) = f(xy)$, then h is differentiable at $\left(d, \dfrac{c}{d}\right)$ for all $d \neq 0$.

Problem 4. Let $R \subseteq \mathbb{R}^2$ be an open region, and $f : R \to \mathbb{R}$ be a function of two variables. Suppose that the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ are bounded on R; that is, there exists a real number $M > 0$ such that

$$\left|\frac{\partial f}{\partial x}(x,y)\right|, \left|\frac{\partial f}{\partial y}(x,y)\right| \leqslant M \qquad \forall\,(x,y) \in R\,.$$

Show that f is continuous on R.