Calculus MA1001-A Final Exam Sample

National Central University, Jan. 7, 2018

Problem 1. Show that $\int_1^e (\ln x)^n dx = e \sum_{k=0}^{n-2} (-1)^k \frac{n!}{(n-k)!} + (-1)^{n-1} n!$. (Integration by parts and induction)

Problem 2. For positive integer n, let $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$. Prove that

$$\int_{-1}^{1} P_n(x) P_m(x)\, dx = \begin{cases} 0 & \text{if } m \neq n\,, \\ \dfrac{1}{2n+1} & \text{if } m = n\,. \end{cases}$$

(Integration by parts)

Problem 3. Compute the indefinite integral $\int \tan^4 x\, dx$ using the following methods:

1. Obtain a recurrence relation for the integral of $\int \tan^n x\, dx$ using integration by parts and find the integral. (Integration by parts and induction)
2. Use the substitution $u = \tan x$ and the technique of partial fractions to find the integral. (Integration by substitution and partial fractions)

Problem 4. Find the indefinite integral $\int \left(\frac{\sin x - \cos x}{\sin x + \cos x} \right)^4 dx$. (Techniques of integrations)

Problem 5. Find the indefinite integral $\int x^3 \arcsin x\, dx$ using integration by parts with $u = x^3$ and $dv = \arcsin x\, dx$. (Techniques of integrations)

Problem 6. Find the indefinite integral $\int \frac{x^3 + 2x^2 + 3x - 2}{(x^2 + 2x + 2)^2}\, dx$. (Partial fractions)

Problem 7. Use the substitution $u^3 = \frac{x^3}{1 + x^3}$ and the technique of partial fractions to find the indefinite integral $\int \frac{1}{(1 + x^3)^{\frac{1}{3}}} dx$. (Integration by substitution and partial fractions)

Problem 8. Evaluate $\int_0^{\frac{\pi}{4}} \frac{dx}{2 + \sin(2x)}$. (Integration by substitution and partial fractions)