Calculus MA1001-A Quiz 05

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學號：_____ 姓名：_____

Problem 1. (4pts) Let f, g be defined on an open interval containing c and f, g be differentiable at c. Show that if $g(c) \neq 0$, then

$$\frac{d}{dx}\Big|_{x=c}\left(\frac{f}{g}\right)(x) = \frac{f'(c)g(c) - f(c)g'(c)}{g(c)^2}\,.$$

Problem 2. (3pts) Find the third derivative of $f(x) = \dfrac{1}{1+x^3}$ at $x = 0$.

Solution: Note that $f(x)(1+x^3) = 1$. Therefore, the product rule implies that

$$f'(x)(1+x^3) + 3x^2 f(x) = 0 \Rightarrow f''(x)(1+x^3) + 6x^2 f'(x) + 6xf(x) = 0$$

which further implies that $f'''(x)(1+x^3) + 9x^2 f''(x) + 18xf'(x) + 6f(x) = 0$. Therefore,

$$f''(0) = -6f(0) = -6\,.$$

Problem 3. (3pts) Find the derivative of the function $f(x) = \tan\left[x^2 \sin^2(x^3)\right]$.

Solution: By the chain rule, since $\dfrac{d}{dx}\tan x = \sec^2 x$ and $\dfrac{d}{dx}\sin^2 x = 2\sin x \cos x$,

$$\begin{aligned}
\frac{d}{dx}f(x) &= \sec^2\left[x^2\sin^2(x^3)\right]\cdot\frac{d}{dx}\left[x^2\sin^2(x^3)\right] = \sec^2\left[x^2\sin^2(x^3)\right]\cdot\left[2x\sin^2(x^3) + x^2\frac{d}{dx}\sin^2(x^3)\right] \\
&= \sec^2\left[x^2\sin^2(x^3)\right]\left[2x\sin^2(x^3) + 2x^2\sin(x^3)\cos(x^3)\frac{d}{dx}x^3\right] \\
&= \sec^2\left[x^2\sin^2(x^3)\right]\left[2x\sin^2(x^3) + 6x^4\sin(x^3)\cos(x^3)\right].
\end{aligned}$$