# 國立中央大學數學系博士班資格考(圖論)試題

1. Let G be a graph with minimum degree k. Prove that the following statements are true. (8 points each)
   (a) G contains a cycle of length at least k+1 provided $k \geq 2$.
   (b) If T is an arbitrary tree with k edges, then G contains a subgraph isomorphic to T.
   (c) If G is an outerplanar graph, then $k \leq 2$.
   (d) If G is of diameter 2, then the edge-connectivity of G is equal to k.

2. The following problems are about the edge-coloring of graphs. (8 points each)
   (a) Prove that the complete graph of order n is of Class 1 if and only if n is even.
   (b) Prove that the Petersen graph is of Class 2.
   (c) Prove that any bipartite graph is of Class 1.
   (d) Find infinite many 3-partite graphs which are of Class 2.

3. Prove that the following statements are true. (8 points each)
   (a) In any tournament, there exists a Hamiltonian path.
   (b) In any tournament, there exists a king that is a vertex from which every vertex is reachable by a path of length at most 2.
   (c) There exists a tournament with n vertices which contains at least $n!/2^{n-1}$ distinct Hamiltonian paths.

4. Construct a triangle-free graph G with chromatic number 7. (12 points)