Name:_____________ Id No.:______________________ Class: ___________

Problem 1: (15 points) Show that $f(z) = \frac{\mathrm{Log}(z+5i)}{z^2+3z+2}$ is analytic everywhere except at the points $-1, -2$, and on the ray $\{(x,y)|x \leq 0, y = -5\}$.

Problem 2:

(1) **(10 points)** Show that $\tanh^{-1} z = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$.

(2) **(5 points)** Compute $\tanh^{-1}(1+2i)$.

Problem 3:(10 points) Determine whether $\log(i^2)$ and $2\log i$ are equal or not when the branch $\log z = \ln r + i\theta (r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4})$ is used. Show your work.

Problem 3: (15 points) Consider $I = \int_C \frac{1}{z^3(z+4)} dz$.

(1) Evaluate the integral I when the contour C is the positively oriented circle $|z| = 2$.

(2) Evaluate the integral I when the contour C is the positively oriented circle $|z+2| = 3$.

Problem 4: (10 points) Consider the function $f(z) = (z-2)^4$ and the closed rectangular region R with vertices at $0, -1, -1+4i, 4i$. Find the points on or inside R at which $|f(z)|$ attains its maximum and minimum values.

Problem 5: (10 points) Let f be an entire function with the property that $|f(z)| \geq 1$ for all z. Show that f is constant.

Problem 6: (15 points) Compute $\int_C \frac{1}{z^2-z} dz$, where C is a line segment from 2 to $2+i$.

Problem 7: (10 points) Let C_R denote the upper half of the circle $|z| = R (R > 2)$, taken in the counterclockwise direction. Use ML bounds to find an upper bound on the modulus of the following contour integral $\int_{C_R} \frac{2z^2-1}{z^4+5z^2+4} dz$.