Name:____________ Id No.:______________________ Class: __________

Problem 1: (10 points)

(1) Compute $(\sqrt{3}+i)^6$.

(2) Find the principal argument $\operatorname{Arg} z$ when $z = \frac{-2}{1+\sqrt{3}i}$.

Problem 2: (15 points) Determine and sketch the set of points determined by

$$| z - 2 | > | z - 3 | .$$

Problem 3: (15 points) Find the cube roots of $-i$ in exponential form and also in rectangular coordinates, exhibit them as vertices of a certain regular polygon, and identify the principal root.

Problem 4: (15 points) Let $f(z) = \left(\frac{z}{\bar{z}}\right)^2$ for $z \neq 0$. Determine whether or not the limit of $f(z)$ exists as $z \to 0$. If so, find the limit. If not, explain the reason carefully.

Problem 5: (15 points) Determine where the following function

$$f(z) = x^3 + 3xy^2 + i(y^3 + 3x^2 y)$$

is differentiable and where it is analytic. Find the values of $f'(z)$ at the points where it is differentiable. Explain the reason carefully.

Problem 6: (15 points) Show that $u(x, y) = xy^3 - x^3 y$ is a harmonic function and find a conjugate harmonic function $v(x, y)$.

Problem 7: (15 points)

(1) Let $z = x + iy$ and $w = u + iv$. Find the images of $x = 1$ and $y = 2$ on the complex w-plane under the mapping $w = f(z) = z^2$. You must write your answers as functions of u and v only.

(2) Sketch the image of the rectangle $\{(x, y) : 0 < x < 1, 0 < y < 2\}$ on the complex w-plane under the mapping $w = f(z) = z^2$.