Complex Variables I — Midterm 2

Name:____________ Id No.:______________________ Class: __________

Problem 1: (10 points) Determine all of the values of z at which the following function is analytic $f(z) = \frac{\operatorname{Log}(z+3)}{z^2+i}$.

Problem 2: (10 points) Solve the equation $\cos z = \sqrt{2}$ for z.

Problem 3:

(1) **(15 points)** Show that $\log(i^{1/2}) = \frac{1}{2}\log i$.

(2) **(5 points)** Find the principal value of $[\frac{e}{2}(-1-\sqrt{3}i)]^{3\pi i}$.

Problem 4: (15 points) Consider $I = \int_C \frac{\cos z}{(z+\pi)^5} dz$.

(1) Evaluate the integral I when the contour C is the square whose edges lie along the lines $x = \pm 4$ and $y = \pm 4$ with positive orientation.
(2) Evaluate the integral I when the contour C is the square whose edges lie along the lines $x = \pm 1$ and $y = \pm 1$ with positive orientation.

Problem 5: (10 points) Find the maximum and minimum moduli of $z^2 - z$ in the disc: $|z| \leq 1$.

Problem 6: (15 points) Let C be the positively oriented circle $\{|z| = 2\}$. Evaluate the contour integral

$$I = \int_C \frac{\cos(\pi z)}{z(z-1)} dz.$$

Problem 7: (10 points) Evaluate the contour integral

$$I = \int_C z^{-1+i} dz$$

where the branch is defined by $z^{-1+i} = e^{(-1+i)\log z}$ ($|z| > 0$, $0 < \arg z < 2\pi$) and C is the positively oriented unit circle $|z| = 1$.

Problem 8: (10 points) Use ML inequality to show that

$$\Big| \int_C \frac{(z^2+3)e^{iz} \operatorname{Log} z}{z^2-2} dz \Big| \leq \frac{7(3\ln 2 + \pi)\pi}{9},$$

where C is the contour $\{z | z = 2e^{i\theta}, 0 \leq \theta \leq \frac{\pi}{3}\}$.