NAME:____________ ID NO.:______________________ CLASS: __________

Problem 1:

(1) (5 points) Compute $(\sqrt{3} - i)^{-10}$.

(2) (5 points) Find the principal argument $\operatorname{Arg} z$ when $z = \frac{2i}{-\sqrt{2}+\sqrt{2}i}$.

Problem 2: Determine and **sketch** the set of points determined by

(1) (8 points) $| z - i | > | z + 1 |$.

(2) (7 points) $\{z : | \operatorname{Arg} z - \pi/2 | < \pi/2\}$.

Problem 3: Find all of the roots of $(-32)^{1/5}$

(1) (5 points) in exponential form.

(2) (5 points) in rectangular coordinates.

(3) (5 points) exhibit them as vertices of a certain regular polygon.

Problem 4: (15 points) Let $f(z) = \frac{\operatorname{Im}(z^2)}{|z|^2}$ for $z \neq 0$. Determine whether or not the limit of $f(z)$ exists as $z \to 0$. If so, find the limit. If not, explain the reason carefully.

Problem 5: Let $f(z) = e^{iz^2} = e^{-2xy}e^{i(x^2-y^2)}$.

(1) (2 points) Write $f(z)$ in rectangular coordinates.

(2) (10 points) Show that $f(z)$ is an entire function.

(3) (3 points) Find $f'(z)$ as a function of z.

Problem 6: Let $u(x, y) = e^{-x} \cos y + xy$.

(1) (5 points) Show that $u(x, y)$ is a harmonic function

(2) (10 points) Find $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is analytic and $f(0) = 1$.

(3) **Bonus problem** (5 points) Find $f(z)$ as a function of z.

Problem 7: Let $z = x + iy$ and $w = u + iv$. Find the images of the mapping $w = z^2$ in each case.

(1) (5 points) ~~The region $\{(x, y) : x \leq 2\}$. You must write your answer as a function of u and v only.~~

(2) (5 points) The region $\{(x, y) : x \geq y\}$.

(3) (5 points) Sketch the image of the triangle with vertices $0, 2,$ and $2 + 2i$ on the complex w-plane.