最佳化方法與應用 MA5038

Homework Assignment 2

Due Jun. 03. 2024

Problem 1. Prove the second part of Theorem 17.4 (in the textbook or the theorem on page 101 of the slide). That is, if $\hat{x}$ is a stationary point of the ℓ_1 penalty function $\phi_1(\cdot;\mu)$ for all μ sufficiently large, but $\hat{x}$ is infeasible for problem

$$\min_x f(x) \quad \text{subject to} \quad \begin{cases} c_i(x) = 0, i \in \mathcal{E}, \\ c_i(x) \geqslant 0, i \in \mathcal{I}, \end{cases}$$

then $\hat{x}$ is an infeasible stationary point.

Hint: Use the fact that $D(\phi_1(\hat{x};\mu);p) = (\nabla f)(\hat{x})^{\mathrm{T}} p + \mu D(h(\hat{x});p)$, where h is defined by

$$h(x) = \sum_{i\in\mathcal{E}} |c_i(x)| + \sum_{i\in\mathcal{I}} [c_i(x)]^-.$$

Problem 2. Verify that the KKT conditions for the bound-constrained problem

$$\min_{x\in\mathbb{R}^n} \phi(x) \qquad \text{subject to} \qquad \ell \leqslant x \leqslant u$$

are equivalent to the compactly stated condition

$$x - \mathrm{P}(x - \nabla\phi(x), \ell, u) = 0,$$

where the projection operator P onto the rectangular box $[\ell, u]$ is defined by

$$P(g,\ell,u)_i = \begin{cases} \ell_i & \text{if } g_i \leqslant \ell_i, \\ g_i & \text{if } g_i \in (\ell_i, u_i), \\ u_i & \text{if } g_i \geqslant u_i, \end{cases} \qquad \text{for all } i = 1,\, 2,\, \cdots,\, n.$$

Problem 3. 請閱讀投影片 173-176 頁（以了解為何要問接下來的問題）。證明給定 x, λ^k 與 μ_k 時，最大值問題

$$\max_{\lambda\geqslant 0} \left\{ f(x) - \sum_{i\in\mathcal{I}} \lambda_i c_i(x) - \frac{1}{2\mu_k} \sum_{i\in\mathcal{I}} (\lambda_i - \lambda_i^k)^2 \right\}$$

在滿足

$$\lambda_i = \lambda_i(x) \equiv \begin{cases} 0 & \text{if } -c_i(x) + \lambda_i^k/\mu_k \leqslant 0; \\ \lambda_i^k - \mu_k c_i(x) & \text{otherwise.} \end{cases}$$

之 λ 取到最大值。