Exercise Problem Sets 4

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Problem 1. Solve $\boldsymbol{X}' = \boldsymbol{A}(t)\boldsymbol{X} + \boldsymbol{F}(t)$ with the following $\boldsymbol{A}$ and $\boldsymbol{F}$.

1. $\boldsymbol{X}' = \begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} \sin 2t \\ \cos 2t \end{bmatrix} e^{2t}.$
2. $\boldsymbol{X}' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} \sec t \\ 0 \end{bmatrix}.$
3. $\boldsymbol{X}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} 0 \\ \sec t \tan t \end{bmatrix}.$
4. $\boldsymbol{X}' = \begin{bmatrix} 1 & 2 \\ -\frac{1}{2} & 1 \end{bmatrix} \boldsymbol{X} + \begin{bmatrix} \csc t \\ \sec t \end{bmatrix} e^{t}.$

Problem 2. Solve $\boldsymbol{X}' = \boldsymbol{A}\boldsymbol{X}$ using the matrix exponential for the following $\boldsymbol{A}$. In each of the following problems, two matrices $\boldsymbol{A}$ and $\boldsymbol{\Lambda}$ will be given, and there exists $\boldsymbol{P}$ such that $\boldsymbol{A} = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{P}^{-1}$. Try to find $\boldsymbol{P}$ and then find the general solution to the linear system $\boldsymbol{X}' = \boldsymbol{A}\boldsymbol{X}$.

1. $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$
2. $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
3. $\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 0 & -2 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$
4. $\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 1 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$
5. $\boldsymbol{A} = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 3 & 1 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$
6. $\boldsymbol{A} = \begin{bmatrix} 4 & 1 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 & 0 \\ 8 & 4 & 0 & 1 & 0 \\ 16 & 8 & -4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \boldsymbol{\Lambda} = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}.$