

Exercise Problem Sets 7

Jun. 18 2020
(due Jun. 24 2020)

Problem 1. Consider the following one-dimensional convection-diffusion problem:

$$\begin{cases} -\varepsilon u''(x) + u'(x) = 0 & \text{for } x \in (0, 1), \\ u(0) = 1, u(1) = 0. \end{cases} \quad (\star)$$

Write the computer codes for numerical solution of problem (\star) by using the finite difference methods on the uniform mesh of $[0, 1]$ with mesh size h :

1. Replace $u''(x_i) \approx \frac{U_{i+1} - 2U_i + U_{i-1}}{h^2}$ and $u'(x_i) \approx \frac{U_{i+1} - U_{i-1}}{2h}$ and consider $(\varepsilon, h) = (0.01, 0.1)$, $(\varepsilon, h) = (0.01, 0.01)$. Plot u_h .
2. Replace $u''(x_i) \approx \frac{U_{i+1} - 2U_i + U_{i-1}}{h^2}$ and $u'(x_i) \approx \frac{U_i - U_{i-1}}{h}$ (upwinding) and consider $(\varepsilon, h) = (0.01, 0.1)$, $(\varepsilon, h) = (0.01, 0.01)$. Plot u_h .