

## Exercise Problem Sets 5

Jun. 04 2020  
(due Jun. 10 2020)

**Problem 1.** Consider solving the initial value problem

$$x' = f(t, x) \equiv -15x, \quad x(0) = 1$$

using numerical method. Note that the exact solution to the IVP above is  $x(t) = e^{-15t}$ . Complete the following.

- (1) Use the forward/explicit Euler method

$$x_{n+1} = x_n + hf(t_n, x_n)$$

with  $h = 1/4, 1/8, 1/16, 1/32, 1/64$  and  $1/128$  to compute approximated values of  $x(1)$ . Plot the numerical solution versus the exact solution.

- (2) Find the global truncation error at  $t = 1$  and maximal local truncation errors for each  $h$ . Examine numerically that the forward/explicit Euler method is an order one method.

- (3) Use the backward/implicit Euler method

$$x_{n+1} = x_n + hf(t_{n+1}, x_{n+1})$$

with  $h = 1/4, 1/8, 1/16, 1/32, 1/64$  and  $1/128$  to compute approximated values of  $x(1)$ . Plot the numerical solution versus the exact solution.

- (4) Find the global truncation error at  $t = 1$  and maximal local truncation errors for each  $h$ . Examine numerically that the backward/implicit Euler method is an order one method.