# Exercise Problem Sets 2

Apr. 2 2020

(due Apr. 8. 2020)

**Problem 1.** In this problem you are asked to write a computer code of a function

$$\text{Div\_Diff:} \quad \begin{array}{ccc} \mathbb{R}^{n+1} \times \mathbb{R}^{\text{n+1}} & \to & \mathbb{R}^{n+1} \\ (\boldsymbol{x}, \boldsymbol{y}) & \mapsto & \boldsymbol{c} \end{array} \quad \text{in the format } \boldsymbol{c} = \text{Div\_Diff}(\boldsymbol{x}, \boldsymbol{y}),$$

where

1. the inputs $\boldsymbol{x}$, $\boldsymbol{y}$ are vectors of length $n+1$ given by $\boldsymbol{x} = [x_0, x_1, \cdots, x_n]$ and $\boldsymbol{y} = [y_0, y_1, \cdots, y_n]$, and
2. the output $\boldsymbol{c}$ is a vector of length $n+1$ given by

$$\boldsymbol{c} = [c_0, c_1, \cdots, c_n] \,,$$

where $c_i$'s are the coefficients of the $n$-th Lagrange interpolating polynomial $p$ satisfying

$$p(x) = c_0 + \sum_{i=1}^{n} c_j (x - x_0)(x - x_1) \cdots (x - x_{i-1}) \,.$$

Note that when $p$ is the $n$-th Lagrange interpolating polynomial for the data $\big(x_0, f(x_0)\big)$, $\big(x_1, f(x_1)\big)$, $\cdots$, $\big(x_n, f(x_n)\big)$, the coefficient $c_i$ is the divided difference $f[x_0, x_1, \cdots, x_i]$.

**Hint**: The algorithm for computing the divided difference $f[x_0, x_1, \cdots, x_n]$ is stated below:

**INPUT**: numbers $x_0, x_1, \cdots, x_n$; values $f(x_0), f(x_1), \cdots, f(x_n)$ as $F_{0,0}$, $F_{1,0}$, $\cdots$, $F_{n,0}$.

**OUTPUT**: the numbers $F_{0,0}$, $F_{1,1}$, $\cdots$, $F_{n,n}$, where $F_{i,i} = f[x_0, x_1, \cdots, x_i]$ satisfies

$$P_n(x) = F_{0,0} + \sum_{i=1}^{n} F_{i,i} \prod_{j=0}^{i-1} (x - x_j) \,,$$

by the following steps:

Step 1 For $i = 1, 2, \cdots, n$

For $j = 1, 2, \cdots, i$

set $F_{i,j} = \dfrac{F_{i,j-1} - F_{i-1,j-1}}{x_i - x_{j-1}}$. $\quad \Big(F_{i,j} = f[x_{i-j}, x_{i-j+1}, \cdots, x_i]\Big)$

Step 2 Output $(F_{0,0}, F_{1,1}, \cdots, F_{n,n})$;

STOP.