Calculus MA1002-B

National Central University, Apr. 23 2024

第三次加分作業

Problem 1. Suppose that $w = f(x, y, z)$ is a differential function on $\mathbb{R}^3$, and

$$F(\rho, \theta, \varphi) = f(\rho\cos\theta\sin\varphi, \rho\sin\theta\sin\varphi, \rho\cos\varphi)\,.$$

1. Show that

$$\begin{aligned} f_x &= F_\rho\cos\theta\sin\phi - F_\theta\frac{\sin\theta}{\rho\sin\phi} + F_\phi\frac{\cos\theta\cos\phi}{\rho}\,,\\ f_y &= F_\rho\sin\theta\sin\phi + F_\theta\frac{\cos\theta}{\rho\sin\phi} + F_\phi\frac{\sin\theta\cos\phi}{\rho}\,,\\ f_z &= F_\rho\cos\phi - F_\phi\frac{\sin\phi}{\rho}\,, \end{aligned}$$

   whenever the denominator is non-zero.

2. Express $|\nabla f|^2$ in terms of F_ρ, F_θ and F_ϕ.