# Calculus MA1002-B Midterm 2 Sample

National Central University, Apr. 11, 2019

**Problem 1.** 定義、定理敘述與證明題

**Problem 2.** Use the root test to determine the whether the series $\sum\limits_{n=0}^{\infty} \dfrac{(3n)!}{n!(n+1)!(n+2)!}$ converges or not.

**Problem 3.** Find all $p \in \mathbb{R}$ such that $\sum\limits_{k=3}^{\infty} \dfrac{\ln(1+k) - \ln k}{(\ln k)^p \ln(\ln k)}$ converges. Note that you need to provide the reason for the convergence or divergence of the power series for each $p$.

**Problem 4.** Show that $\sum\limits_{k=1}^{\infty} \dfrac{(-1)^k \cos(kx)}{k}$ converges for all $x \in \mathbb{R}$.

**Problem 5.** Find the interval of convergence of the power series

$$\sum_{k=2}^{\infty} \frac{x^{2k}}{k2^k (\ln k)^2} \,.$$

**Problem 6.** Find the power series solution $y(x) = \sum\limits_{n=0}^{\infty} a_n x^n$ of the differential equation

$$x^2 y''(x) + xy'(x) + (x^2 - 1)y(x) = 0 \,, \qquad y(0) = 0 \,, \quad y'(0) = 1 \,.$$

You need to show all the computations instead of just providing the answer.

**Problem 7.** Find the fourth Maclaurin polynomial for the function $f(x) = e^x \sin^2 x$.

**Problem 8.** Find a natural number $n$ such that

$$\left| \cos 1 - \sum_{k=0}^{n} \frac{(-1)^k}{(2k)!} \right| < 5 \times 10^{-6} \,.$$

Explain your answer.