Calculus MA1001-B Final Exam Sample

National Central University, Jan. 7, 2019

Problem 1. For positive integer n, let $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. Complete the following

1. Show that P_n is a polynomial of degree n.
2. Show that $\frac{d^n}{dx^n}(x^2-1)^m = 0$ at $x = \pm 1$ if $m > n$.
3. Show that $\displaystyle\int_{-1}^{1} P_n(x)P_m(x)\,dx = \begin{cases} 0 & \text{if } m \neq n\,, \\ \frac{1}{2n+1} & \text{if } m = n\,. \end{cases}$ (Integration by parts)

Problem 2. Compute the indefinite integral $\int \sec^3 x\,dx$ using the following methods:

1. Obtain a recurrence relation for the integral of $\int \sec^n x\,dx$ using integration by parts and find the integral. (Integration by parts and induction)
2. Use the substitution $\tan\frac{x}{2} = \frac{1-t}{1+t}$ (without any other substitution of variables) and the technique of partial fractions to find the integral. (Integration by substitution and partial fractions)

Problem 3. Find the indefinite integral $\int x^3 \arcsin x\,dx$ using integration by parts with $u = x^3$ and $dv = \arcsin x\,dx$. (Integration by parts)

Problem 4. Let k be a positive integer. Use the techniques of partial fractions to show that

$$\int \frac{dx}{x^{2k-1}(x^2+1)} = \sum_{j=1}^{k-1} \frac{(-1)^{j+1}}{2j-2k} x^{2j-2k} + (-1)^{k-1}\ln|x| + \frac{(-1)^k}{2}\ln(x^2+1) + C\,.$$

(Partial fractions)

Problem 5. Evaluate $\displaystyle\int_{-1}^{1} (1+4x^2)\ln\frac{\sqrt{5}+2\sqrt{1-x^2}}{\sqrt{1+4x^2}}\,dx$. (Techniques of integrations)

Problem 6. Show that the improper integral $\displaystyle\int_0^\infty x^{\alpha-1}e^{-x}\,dx$ converges for $\alpha > 0$ using the comparison test for improper integrals. (Convergence of the improper integral)