Calculus MA1002-A Midterm 3 Sample

National Central University, May. 25, 2019

Problem 1. 十小題是非題，觀念題，有倒扣。

Problem 2. 定義與定理敘述題。

Problem 3. Assume that f is a continuous function of two variable satisfying that

$$\lim_{(x,y)\to(2,1)} \frac{f(x,y) - 2x^2 + y^3}{\sqrt{(x-2)^2 + (y-1)^2}} = 0\,.$$

1. Find $f_x(2,1)$ and $f_y(2,1)$.
2. Prove or disprove that f is differentiable at $(2,1)$.

(Differentiation of functions of two variables)

Problem 4. Let $f, g : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy(x-y)}{x^2+y^2} & \text{if } (x,y) \neq (0,0)\,, \\ 0 & \text{if } (x,y) = (0,0)\,. \end{cases}$$

Find the directional derivative of f at $(0,0)$ in the direction along which the value of the function f at $(0,0)$ increases most rapidly. **(Directional derivatives)**

Problem 5. Find the second Taylor polynomial of the function $f(x,y) = \arccos \dfrac{x}{\sqrt{x^2+y^2}}$ at $(1,1)$. **(Chain rule and Taylor's polynomial)**

Problem 6. Find all relative extrema and saddle points of $f(x,y) = 2xy - \dfrac{1}{2}(x^4+y^4) + 1$ using the second derivative test. When a relative extremum is found, determine if it is a relative maximum or a relative minimum. **(Relative extrema, saddle points, and the second derivative test)**

Problem 7. Let R be the solid in the space bounded by the paraboloid $z = x^2 + y^2$ and the plane $x + y + z = 4$. Find the extreme value of function $w = f(x,y,z) = xyz$ on R. **(Absolute extrema and Lagrange multipliers)**