# Extra Exercise Problem Set 10

Mar. 29 2019

**Problem 1.**

1. Prove that

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \qquad \forall\, x \in (-1,1)$$

   using the fact that

$$\frac{1-x^{n+1}}{1-x} = 1 + x + x^2 + \cdots + x^n = \sum_{k=0}^{n} x^k \qquad \forall\, x \neq 1\,. \tag{$\star$}$$

2. Use $(\star)$ to show that

$$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}x^k}{k} \qquad \forall\, x \in (-1,1)\,.$$

**Problem 2.** Let $f : (-r, r) \to \mathbb{R}$ be $n$-times differentiable at 0, and $P_n(x)$ be the $n$-th Maclaurin polynomial for $f$.

1. Show that if $g(x) = x^\ell f(x^m)$ for some positive integers $m$ and $\ell$, then the $(mn+\ell)$-th Maclaurin polynomial for $g$ is $x^\ell P_n(x^m)$.

2. Show that if $g(x) = x^\ell f(-x^m)$ for some positive integer $m$ and $\ell$, then the $(mn+\ell)$-th Maclaurin polynomial for $g$ is $x^\ell P_n(-x^m)$.

3. Find the $(2n)$-th Maclaurin polynomial for the function $y = \dfrac{1}{1+x^2}$.