Calculus MA1001-A Quiz 03

National Central University, Oct. 04 2018

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Problem 1. (3pts) Let $f : (a, b) \to \mathbb{R}$ be a real-value function and $a < c < b$. Write down the definition of the continuity of f at c.

Problem 2. (3pts) For what values of a and b is

$$g(x) = \begin{cases} ax + 2b & \text{if } x \leqslant 0 \\ x^2 + 3a - b & \text{if } 0 < x \leqslant 2 \\ 3x - 5 & \text{if } x > 2 \end{cases},$$

continuous at every x?

Solution: For g to be continuous at every point, we need to find a and b so that $\lim\limits_{x \to c} g(x) = g(c)$. Since all the polynomials are continuous function, we find that

$$\lim_{x \to c} g(x) = g(c) \text{ if } c \neq 0, 2.$$

Now, note that $\lim\limits_{x \to 0^-} g(x) = a \cdot 0 + 2b = 2b$; $\lim\limits_{x \to 0^+} g(x) = 0^2 + 3a - b = 3a - b$; $\lim\limits_{x \to 2^-} g(x) = 2^2 + 3a - b = 3a - b + 4$ and $\lim\limits_{x \to 2^+} g(x) = 3 \cdot 2 - 5 = 1$. For g to be continuous at $x = 0$ and $x = 2$, we must have

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} g(x) \quad \text{and} \quad \lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x);$$

thus

$$2b = 3a - b \quad \text{and} \quad 3a - b + 4 = 1.$$

Solving for (a, b) from equations above, we find that $a = b = -\dfrac{3}{2}$.

Problem 3. (4pts) Let $f : [0, 2] \to \mathbb{R}$ be function defined by

$$f(x) = \text{the integer which is nearest to } x.$$

Find all the discontinuities of f. Are they removable or non-removable discontinuities?