Calculus MA1001-A Quiz 02

National Central University, Sept. 27 2018

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Problem 1. (4pts) Write down the definition of the statement $\lim_{x\to\infty} f(x) = L$.

Problem 2. (3pts) Prove the identity $\cos(3x) = 4\cos^3 x - 3\cos x$ for all real numbers x. You might need the sum and difference formula

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \qquad \forall\, x, y \in \mathbb{R}\,.$$

Proof. By the sum and difference formula $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$, we find that

$$\begin{aligned}\cos 3x &= \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x = (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x \\ &= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x = 4\cos^3 x - 3\cos x\,.\end{aligned}$$

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Problem 3. (3pts) Compute the limit $\lim_{x\to 0} \dfrac{\cos(3x) - \cos x}{x^2}$ if it exists.

Solution: If $x \neq 0$, using the formula in Problem 2 we find that

$$\frac{\cos(3x) - \cos x}{x^2} = \frac{4\cos^3 x - 4\cos x}{x^2} = 4\cos x \frac{\cos^2 x - 1}{x^2} = -4\cos x \Big(\frac{\sin x}{x}\Big)^2\,.$$

Since $\lim_{x\to 0} \cos x = 1$ and $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$,

$$\lim_{x\to 0} \frac{\cos(3x) - \cos x}{x^2} = \lim_{x\to 0} \Big[-4\cos x \Big(\frac{\sin x}{x}\Big)^2\Big] = -4\Big(\lim_{x\to 0} \cos x\Big)\Big(\lim_{x\to 0} \frac{\sin x}{x}\Big)^2 = -4\,.$$