Calculus I Midterm 2, Sample

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Problem 1. Find $\dfrac{d}{dx}\displaystyle\int_{2^x}^{\cos^{-1}x} \ln(u^4+1)du$ for $x \in (-1,1)$.

Problem 2. Find $\dfrac{d}{dx}\exp\big[\tan^{-1}\ln(x^2+1)\big]$. Note that $\exp(x) = e^x$.

Problem 3. Find $\displaystyle\lim_{x\to 1^-}\Big(\cot\frac{\pi x}{2}\Big)^{\left(\tan^{-1}x - \frac{\pi}{4}\right)}$.

Problem 4. Find the indefinite integral $\displaystyle\int x^2 \sin^{-1} x dx$. Verify your answer by differentiating the result you obtain.

Problem 5. Find the definite integral $\displaystyle\int_0^{\frac{\pi}{6}} \sec^4 x dx$.

Problem 6. Find the indefinite integral $\displaystyle\int \cos^2 x dx$ using

(1) The half angle formula $\cos^2 x = \dfrac{1+\cos 2x}{2}$;

(2) Using the technique of integration by parts with $u = \cos x$ and $dv = \cos x dx$;

(3) Using the substitution of variable $t = \tan\dfrac{x}{2}$ and transform the original integral into the integral of a rational function, and use the technique of partial fractions.

Hint: For (3), you will need the recursive formula

$$\int \frac{1}{(1+x^2)^n}dx = \frac{x}{2(n-1)(x^2+1)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{1}{(1+x^2)^{n-1}}dx \quad \forall\, n \geq 2\,.$$

Problem 7. The goal of this problem is to find the indefinite integral $\displaystyle\int \frac{1}{(1+x^3)^{\frac{1}{3}}}dx$. Complete the following.

(1) By the substitution of variable $1 + x^{-3} = u^3$, show that

$$\int \frac{1}{(1+x^3)^{\frac{1}{3}}}dx = -\int \frac{u}{u^3-1}du\,.$$

(2) Using the technique of integrating rational functions by partial fractions, find the indefinite integral in (1) and then express the result in terms of x so that one obtains

$$\int \frac{1}{(1+x^3)^{\frac{1}{3}}}dx = -\frac{1}{\sqrt{3}}\tan^{-1}\left[\frac{2(1+x^{-3})^{\frac{1}{3}}+1}{\sqrt{3}}\right] + \frac{1}{6}\ln\left[(1+x^{-3})^{\frac{2}{3}} + (1+x^{-3})^{\frac{1}{3}} + 1\right]$$
$$-\frac{1}{3}\ln\left|(1+x^{-3})^{\frac{1}{3}} - 1\right| + C\,.$$