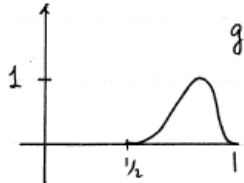
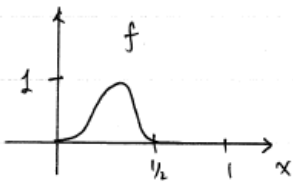


#48) If $\|\cdot\|_\infty = \langle \cdot, \cdot \rangle^{1/2}$ for some inner product $\langle \cdot, \cdot \rangle$ then $\|\cdot\|_\infty$ satisfies

(*) $2\|f\|_\infty^2 + 2\|g\|_\infty^2 = \|f+g\|_\infty^2 + \|f-g\|_\infty^2$. However, this identity cannot hold for all

$f, g \in C_b([0,1], \mathbb{R})$. Take for instance f, g as follows



$$\text{then } \|f\|_\infty = \|g\|_\infty = 1$$

$$\|f+g\|_\infty = 1 = \|f-g\|_\infty$$

The LHS of (*) is then 4 but the RHS of (*) is 2. Hence (*) fails for such f, g .